

Fundamentals of Radio Astronomy

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**Many slides taken from NRAO
Synthesis Imaging Summer School**

**Lectures by Perley, McKinnon, Myers, Carilli:
See: <http://www.aoc.nrao.edu/events/synthesis/2008/>**

Fundamentals of Radio Astronomy

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- **Radio Emission: what can we learn?**
 - Thermal and non-thermal continuum emission, spectral line radiation
 - The radio spectrum (“lo, mid and high”)
 - Interferometers and single dish instruments
 - Science results – what can you do with radio?
- **Antenna Theory & Receivers**
 - Basic description, power patterns and directionality
 - Heterodyne receivers (centimeter-wave)
- **Polarization**
 - Basic concepts and examples; relation to Stokes parameters
- **Interferometry Basics**
 - Getting better resolution
 - Signal multiplication, correlation function and making an image
 - Sensitivity of an interferometer

Fundamentals of Radio Astronomy

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- Equations that you should remember:

- Resolution of an interferometer = $1.02 \lambda/D$ where D = baseline length (i.e., 25 km for the VLA)

- Field of view of an interferometer = primary beam = $1.02 \lambda/D$ where D = dish diameter (i.e., 25 m for the VLA)

- Polarization in terms of Stokes parameters =

$$P = \sqrt{Q^2 + U^2} \quad \tan 2\psi = U / Q$$

- Visibility function and its relation to sky brightness (FT)

$$I_v(l, m) = \frac{1}{N} \sum_{n=1}^N V_n(u_n, v_n) \exp[2\pi i(u_n l + v_n m)] \Delta u \Delta v$$

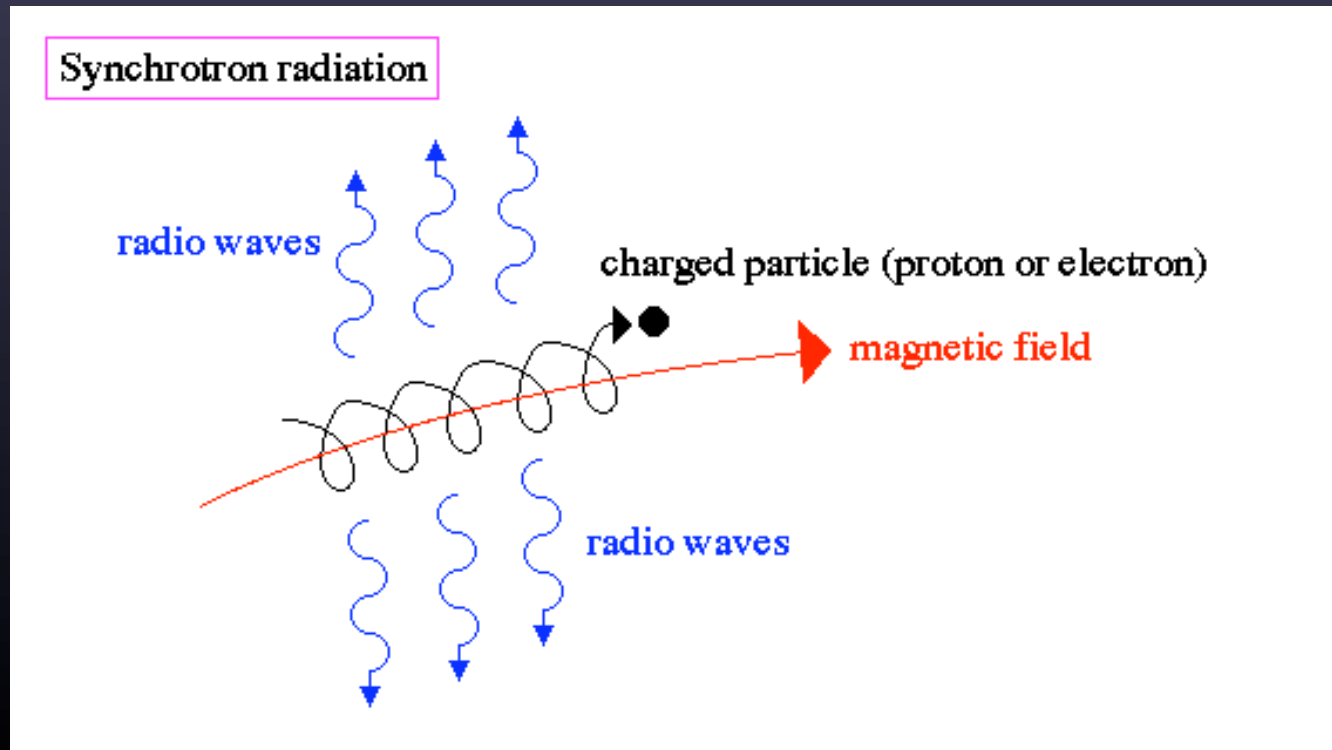
- Sensitivity equation (radiometer equation)

$$S_{rms} = \frac{2kT_{sys}}{A_{eff} \sqrt{N_A (N_A - 1) t_{int} \Delta \nu}}$$

Radio Emission Mechanisms

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- **Synchrotron radiation - continuum**
 - Energetic charged particles accelerating along magnetic field lines (non-thermal)



- What can we learn?

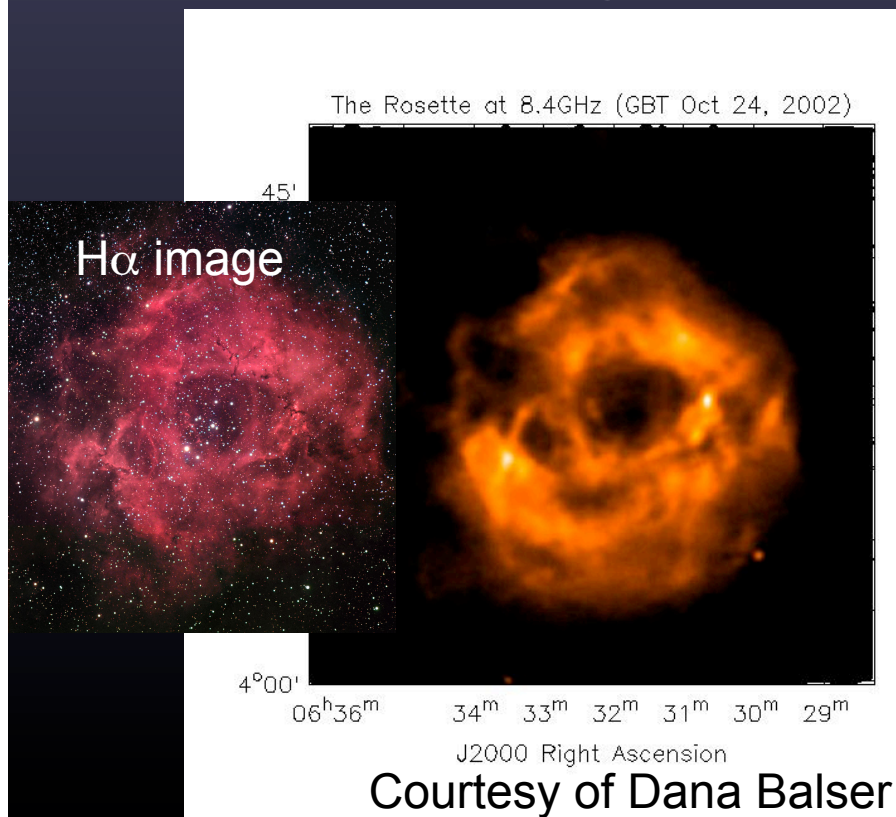
- particle energy
- strength of magnetic field
- polarization
- orientation of magnetic field

Radio Emission Mechanisms

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- **Thermal emission - continuum**

- Blackbody radiation for objects with $T \sim 3-30$ K
- Brehmsstrahlung “free-free” radiation: charged particles interacting in a plasma at T ; e^- accelerated by ion



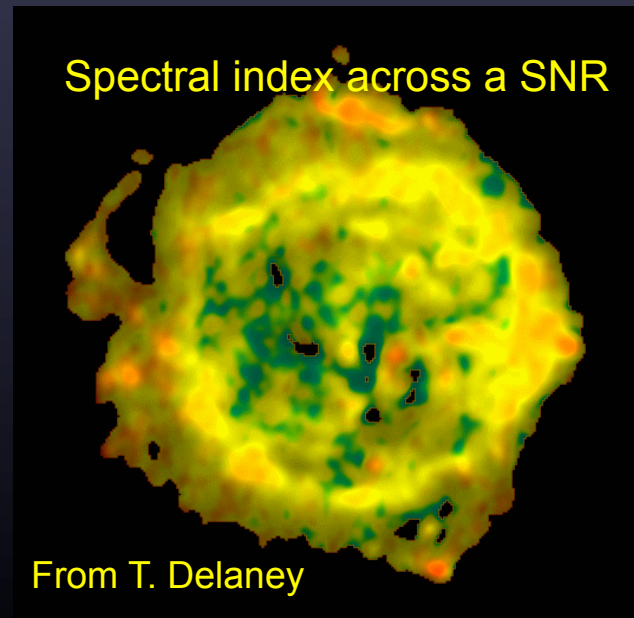
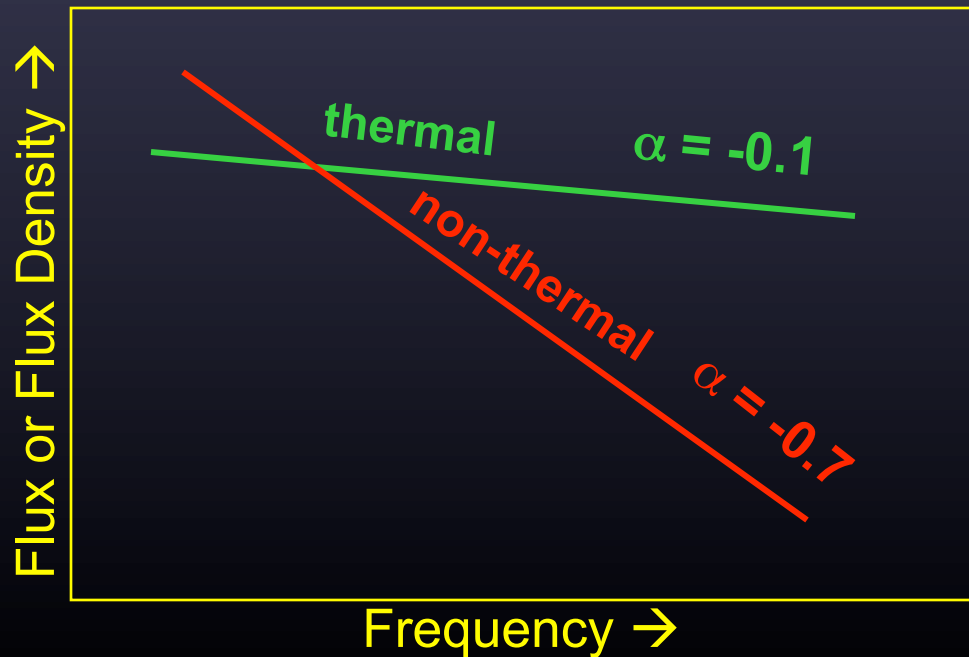
- What can we learn?

- mass of ionized gas
- optical depth
- density of electrons in plasma
- rate of ionizing photons

Radio Emission Mechanisms

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- What we measure from radio continuum
 - Radio flux or flux density at different frequencies
 - Spectral index α , where $S_\nu \sim \nu^\alpha$

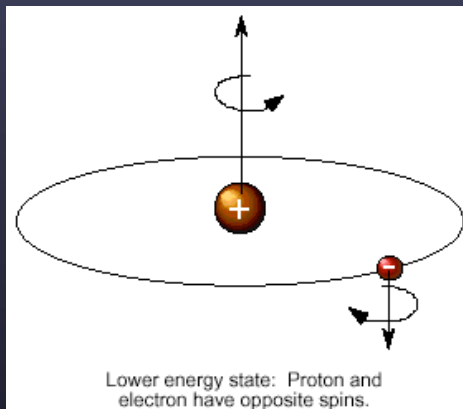


Radio Emission Mechanisms

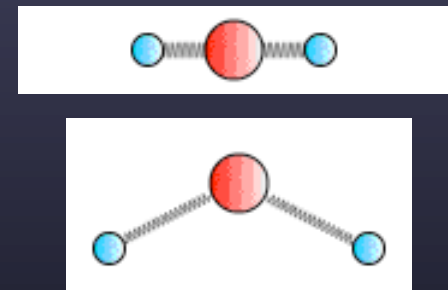
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- Spectral line emission

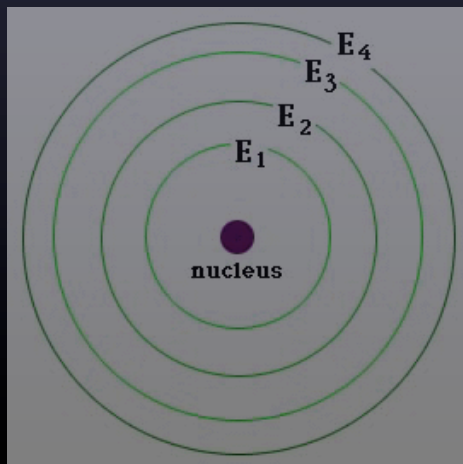
- Discrete transitions in atoms and molecules



Atomic Hydrogen
“spin-flip” transition
21 cm



Molecular Lines
CO, CS, H₂O, SiO, etc.!



Recombination Lines
outer transitions of H
H166 α , H92 α , H41 α
(1.4, 8.3 GHz, 98 GHz)

- What can we learn?

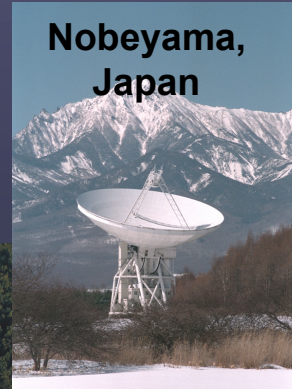
- gas physical conditions (n, T)
- kinematics (Doppler Effect)

A wide variety of single dishes!

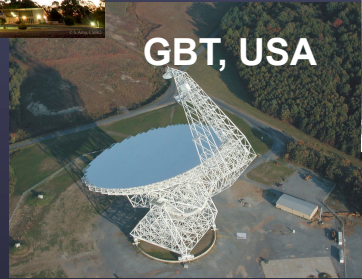
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Parkes, AUS



Nobeyama,
Japan



GBT, USA

Millimeter
> 15 GHz
< 10 mm

Low-Frequency
< 1.4 GHz
> 1 m



IRAM
Spain

Centimeter
1.4 MHz - 15 GHz
20 cm - 1 cm



Arecibo, USA



LMT, Mexico



CSO, Hawaii



Lovell, UK

A wide variety of interferometers!

ATA
CA, USA



VLA/EVLA
NM, USA



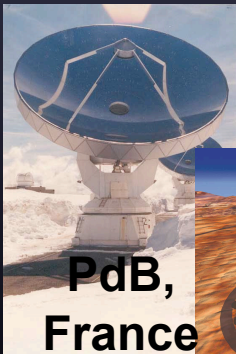
ATCA, Australia



Millimeter
> 15 GHz
< 10 mm

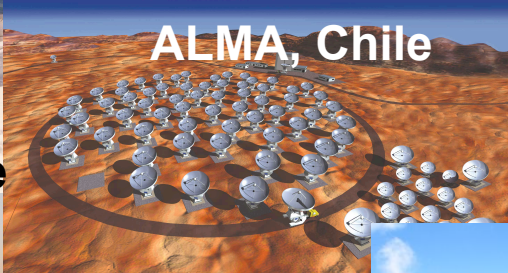
Low-Frequency
< 1.4 GHz
> 1 m

Centimeter
1.4 MHz - 15 GHz
20 cm - 1 cm



PdB,
France

ALMA, Chile



GMRT, India

LOWFAR, NL



CARMA,
CA, USA



SMA, Hawaii, USA



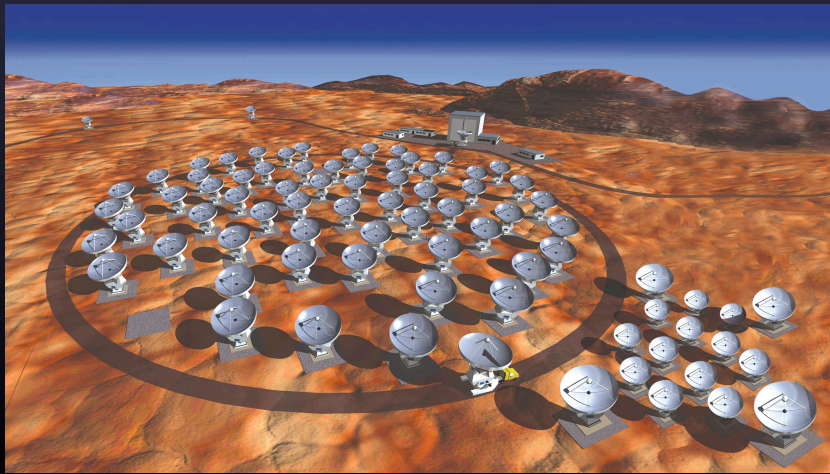
LWA, NM, USA



Resolution of a radio telescope

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- **Resolution of a single dish radio telescope:** $1.02 \lambda/D$ where D = diameter of telescope; therefore, VLA @20 cm: 30' resol.
- **Resolution of an interferometer:** $1.02 \lambda/D$ where D longest baseline: VLA @ 20 cm with 30 km baseline = few arcseconds
- **Primary beam (or “field of view”) for int.** = single dish resol.



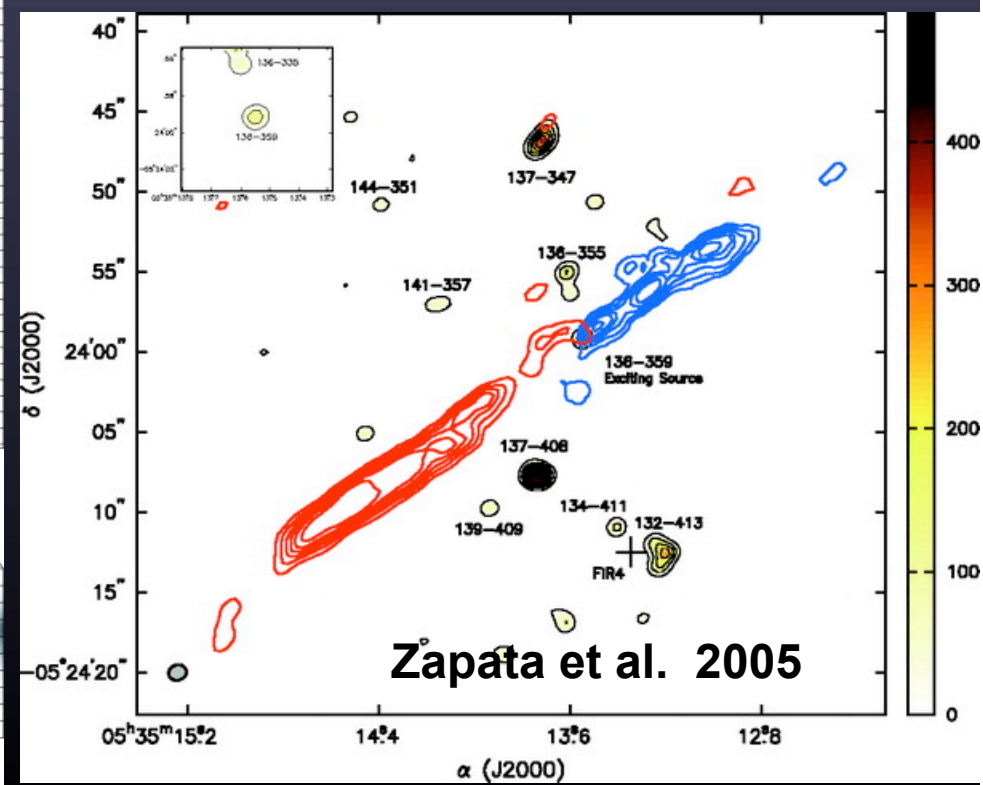
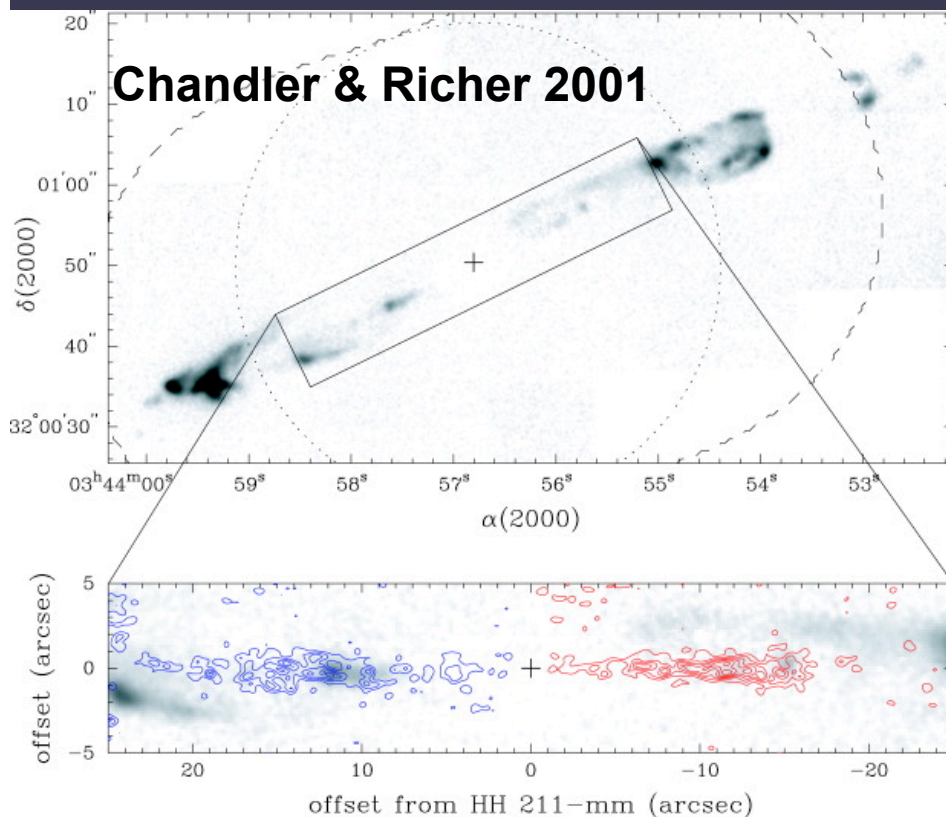
For ALMA, baselines of
Up to 15 km !!

Wavelengths: < 1 mm

Resolutions: < 1”!

Tour of the Galaxy: Interstellar

- Low Mass Star Formation
 - obscured regions of the Galaxy with high resolution
 - collimated outflows powered by protostar – 10000s AU



VLA 7mm spectral line (SiO) – 0.5''

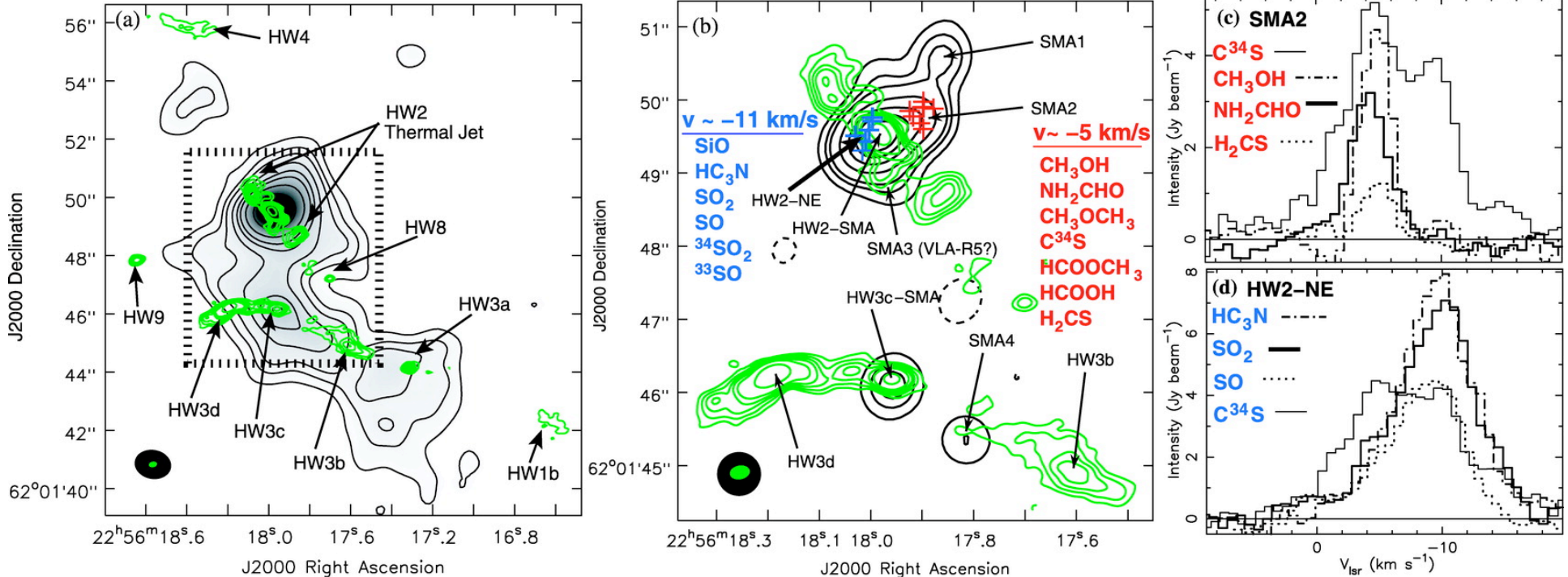
SMA 1mm spectral line (CO 2-1) – 1''

Tour of the Galaxy: Interstellar

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• Probing massive stars in formation

- tend to be forming in clusters; confusion! go to high frequencies (sub-mm)
- “hot molecular cores” (100-300K) around protostars; complex chemistry

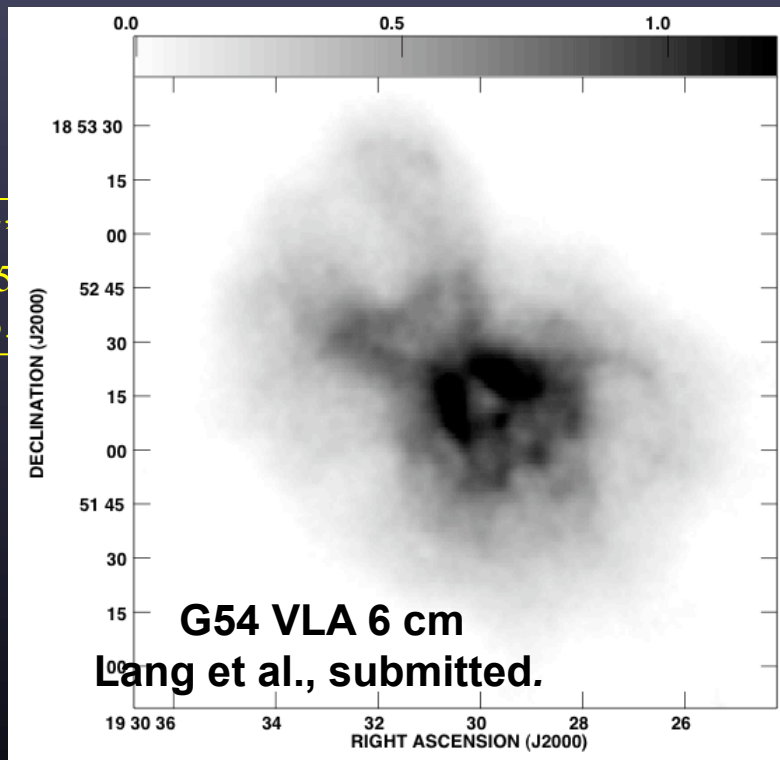


Ceph A-East $d=725$ pc; black=SMA 875 μm ; green=VLA 3 cm; lines=sub-mm species
Spatial resolutions of $<1''$ (where $1'' \sim 0.004$ pc or ~ 750 AU) from Brogan et al. (2007)

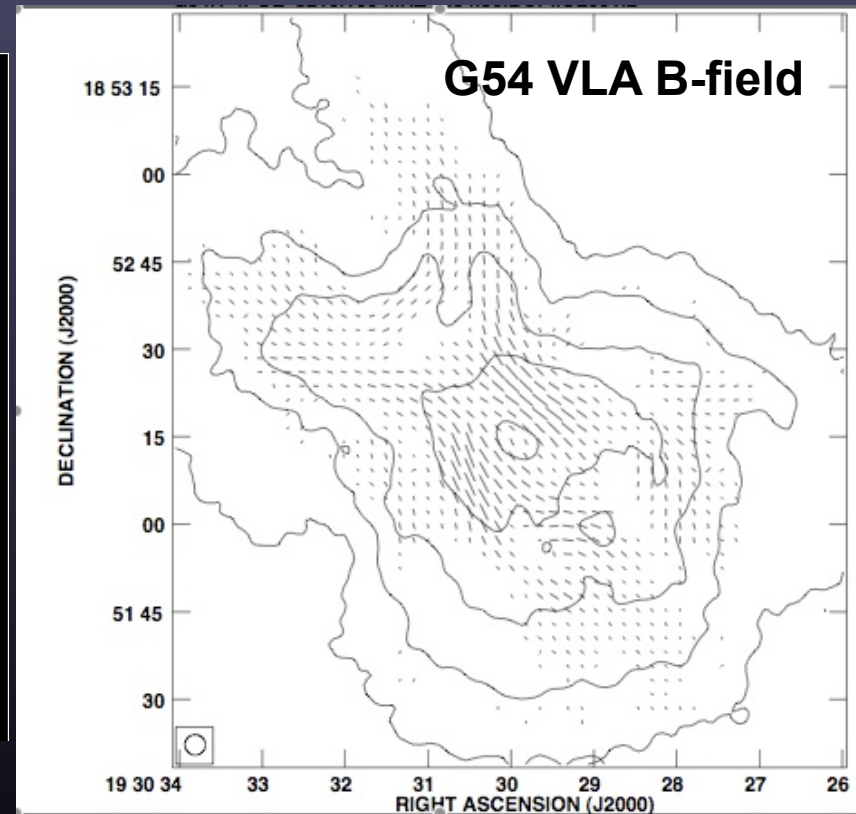
Tour of the Galaxy: Interstellar

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- Star Death: Pulsar Wind Nebulae



2.7
d=5
~ 3



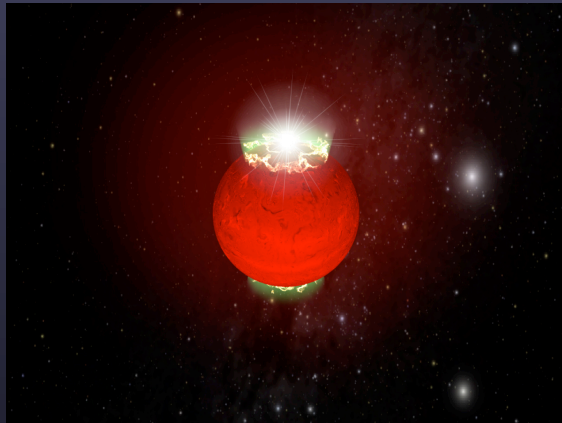
@
2 kpc
1.4 pc

- radio studies: particle energies, polarization, magnetic field orientation
- VLA/VLBA pulsar proper motion can be combined with spin-axis orientation (X-ray)
- Pulsar timing and discovery done with single dish radio telescopes – Parkes, GBT

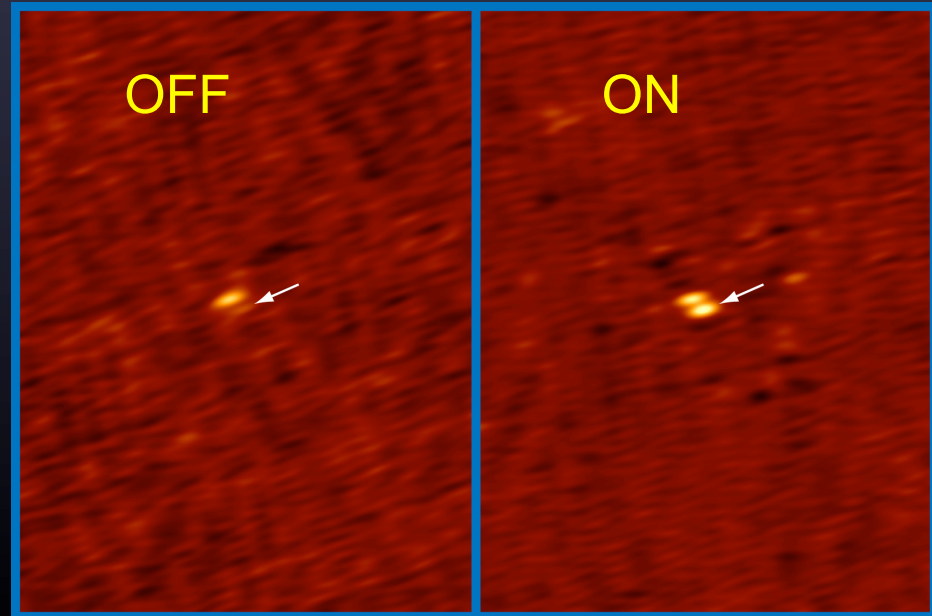
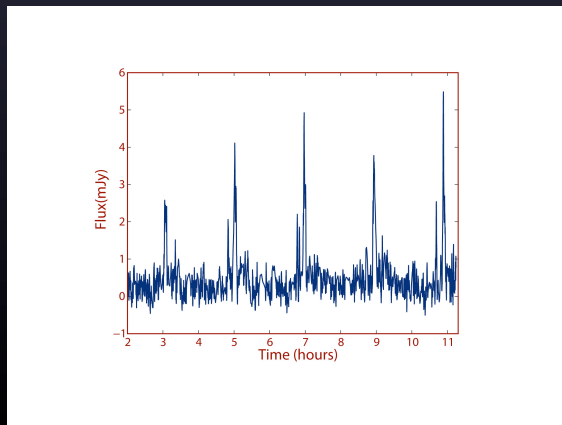
Tour of the Galaxy: Stellar Sources

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- Stars: Very low mass and brown dwarfs
 - some M+L type dwarfs, brown dwarfs show quiescent and flaring non-thermal emission (Berger et al. 2001-7; Hallinan et al. (2006,2008))



<-- magnetic activity at the poles: electrons interact with dwarf's magnetic field to produce radio waves that then are amplified by masers



Tour of the Galaxy: Exotic

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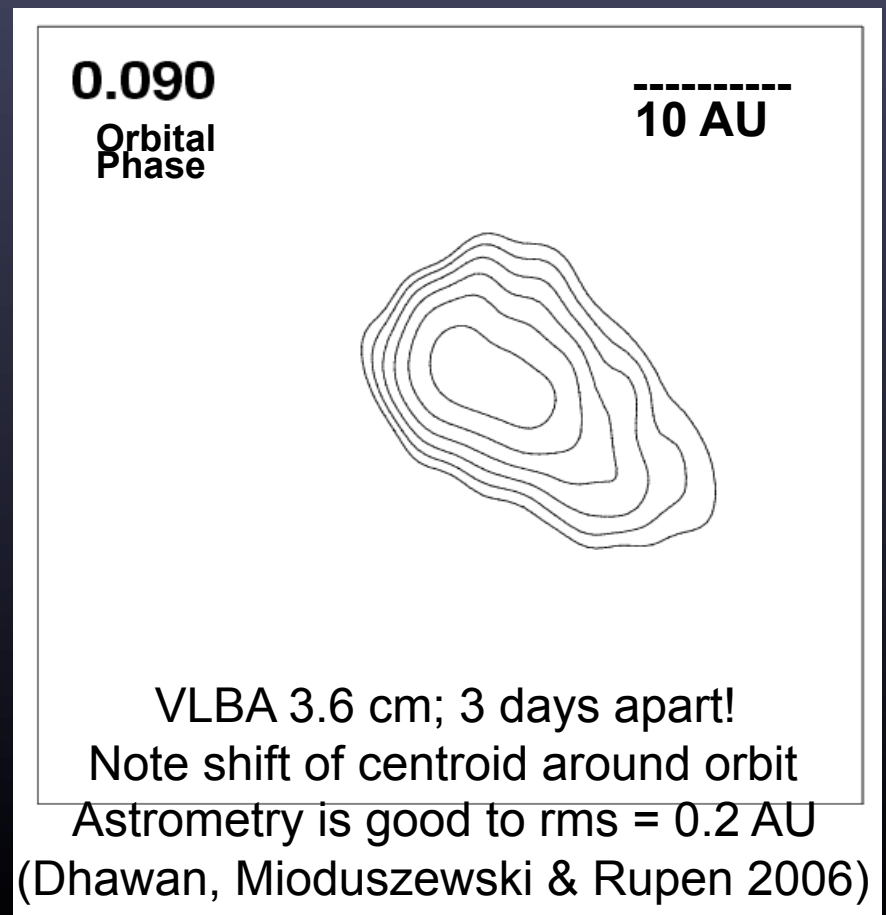
- **LS I+61 303 : A pulsar comet around a hot star?**

- well known radio, X-, γ -ray, source

- high mass X-ray binary with 12 solar mass Be star and NS

- radio emission models:
 - (a) accretion-powered jet or
 - (b) rotation powered pulsar

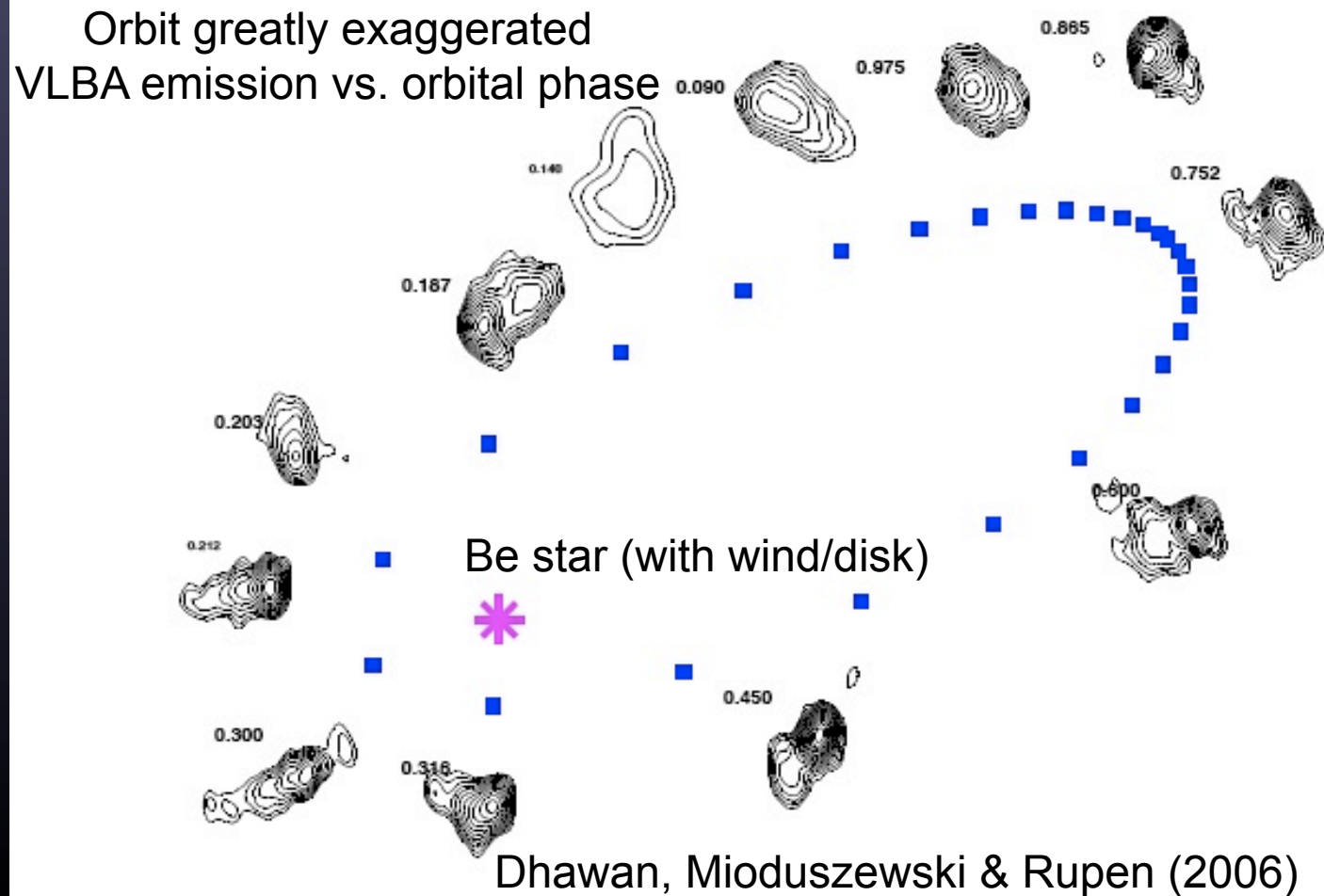
- VLBA data support pulsar model in which particles are shock-accelerated in their interaction with the Be star wind/disk environment



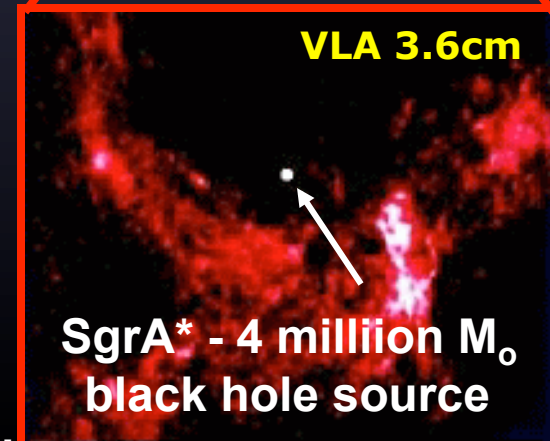
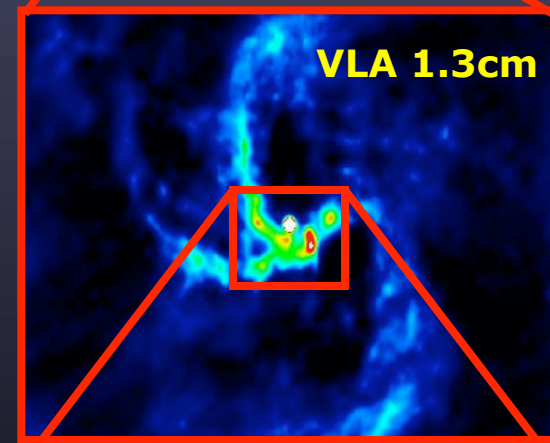
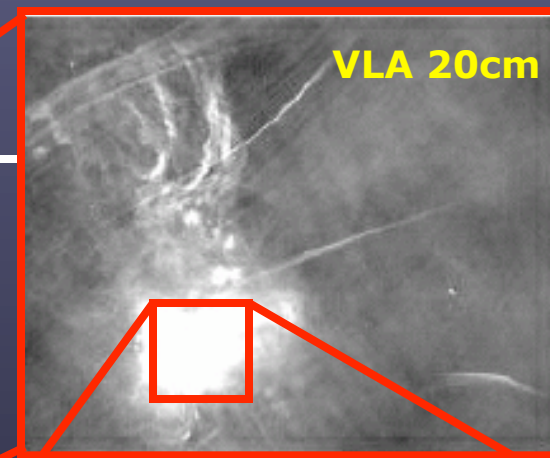
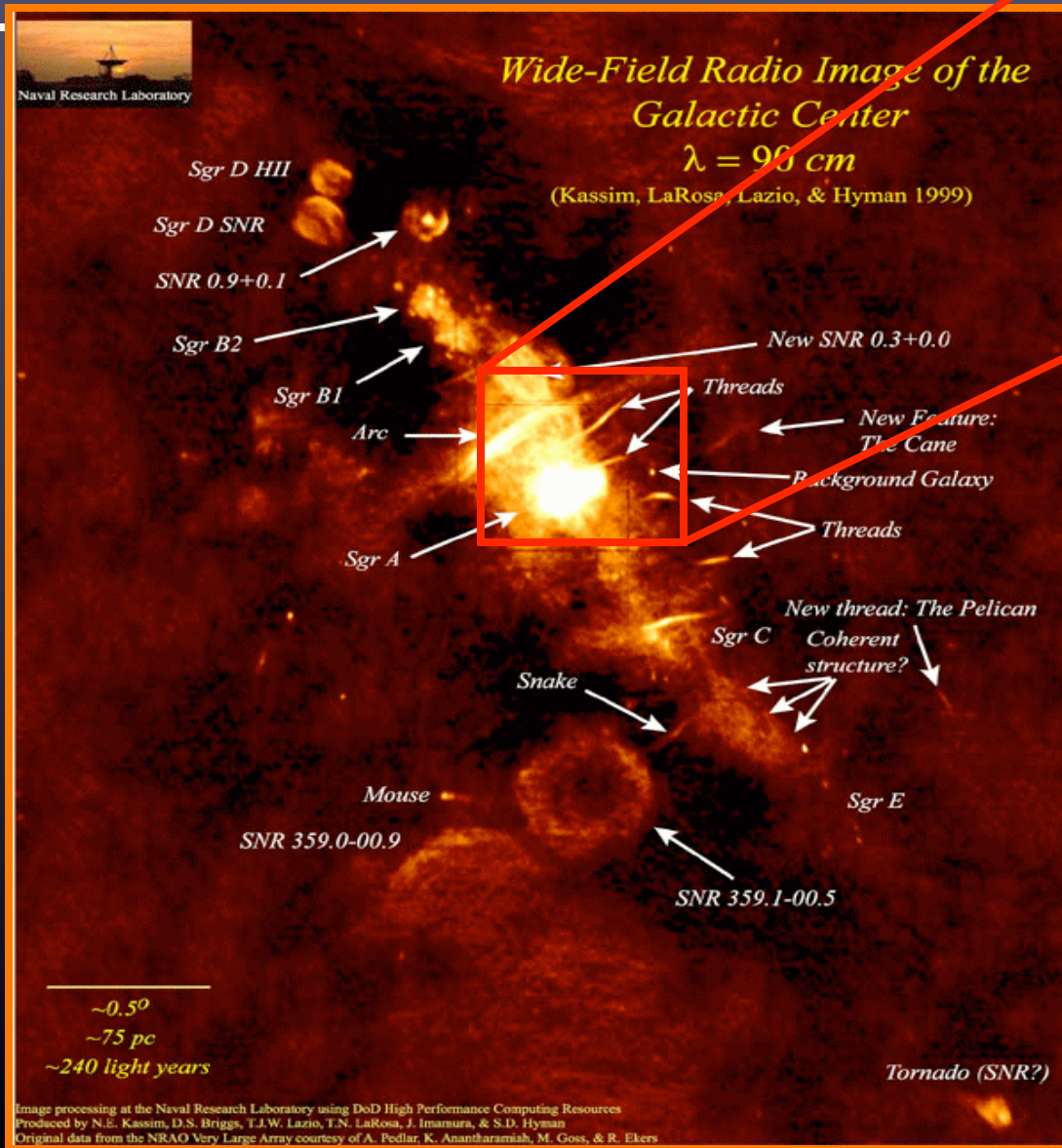
Tour of the Galaxy: Exotic

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- LS I+61 303 : A pulsar comet around a hot star?



Center of our Galaxy

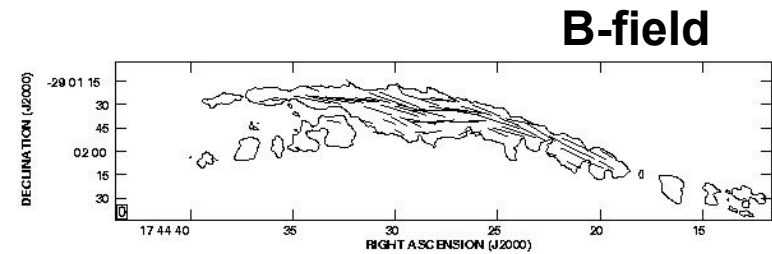
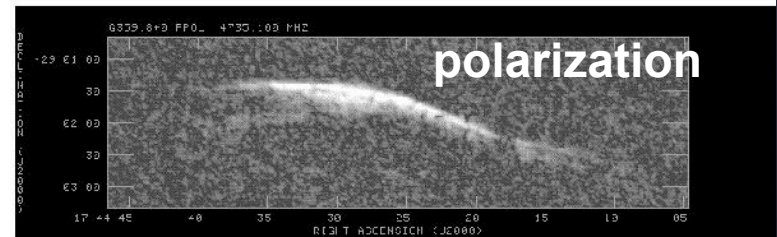
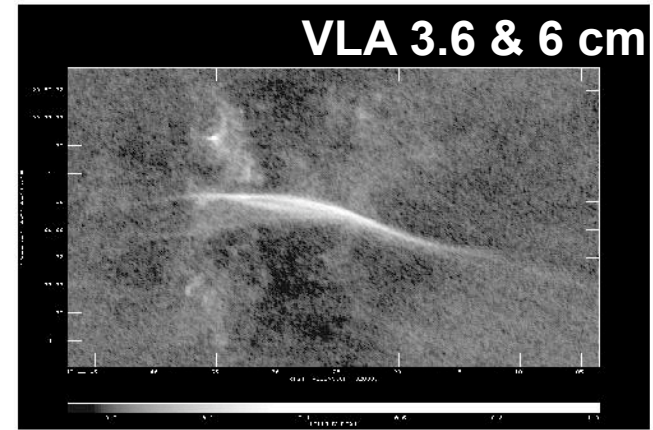
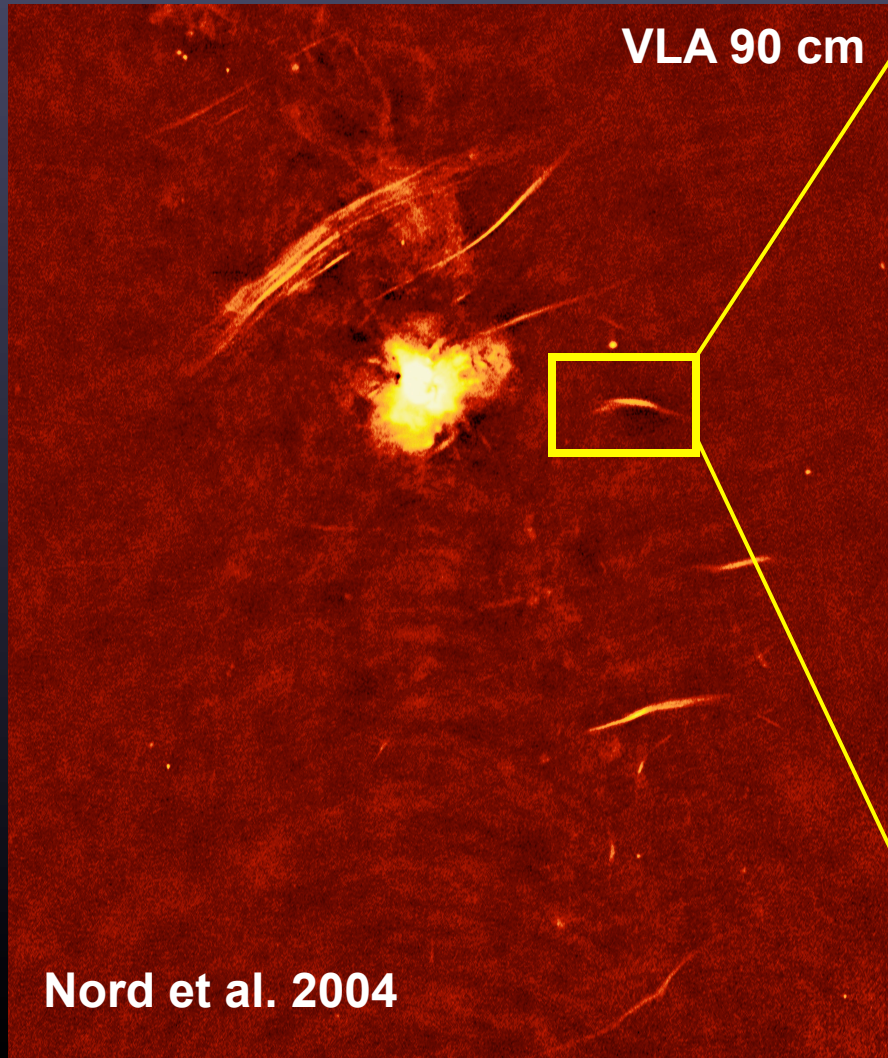


Credits: Lang, Morris, Roberts, Yusef-Zadeh, Goss, Zhao

Tour of the Galaxy: The Galactic Center

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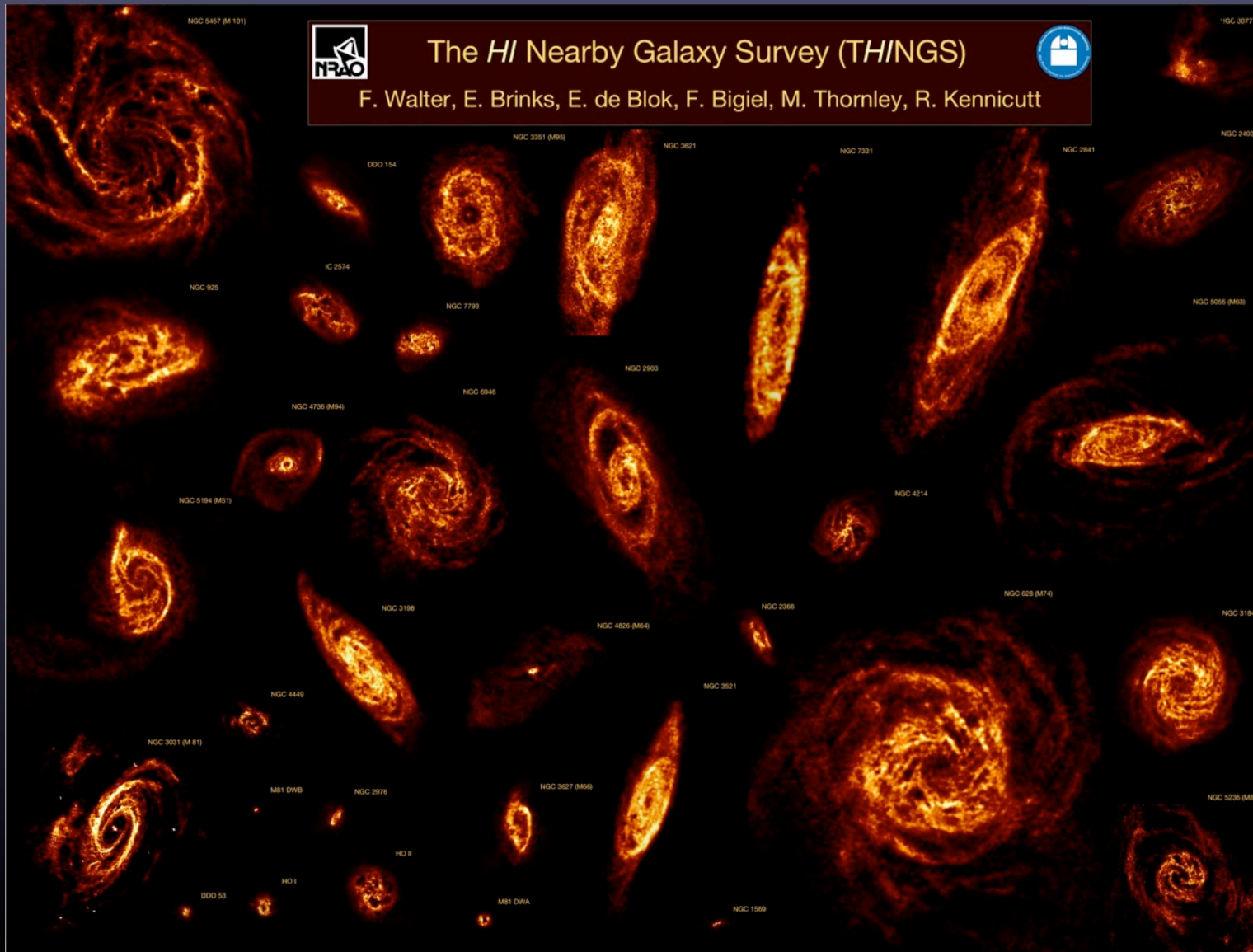
- Magnetic Field: Pervasive vs. Local?



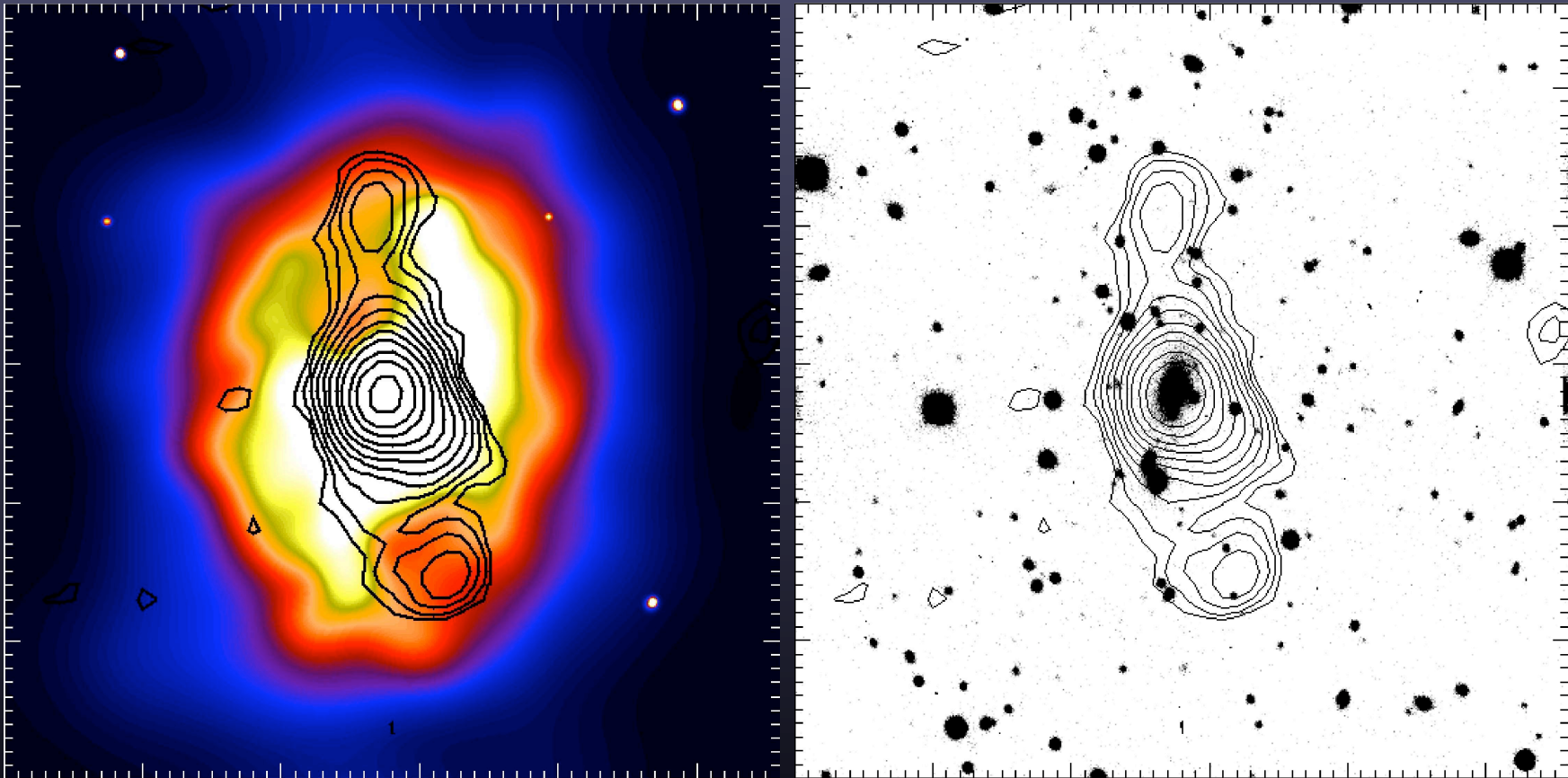
Lang & Anantharamaiah, *in prep.*

Radio Spectral Lines: Cold Gas

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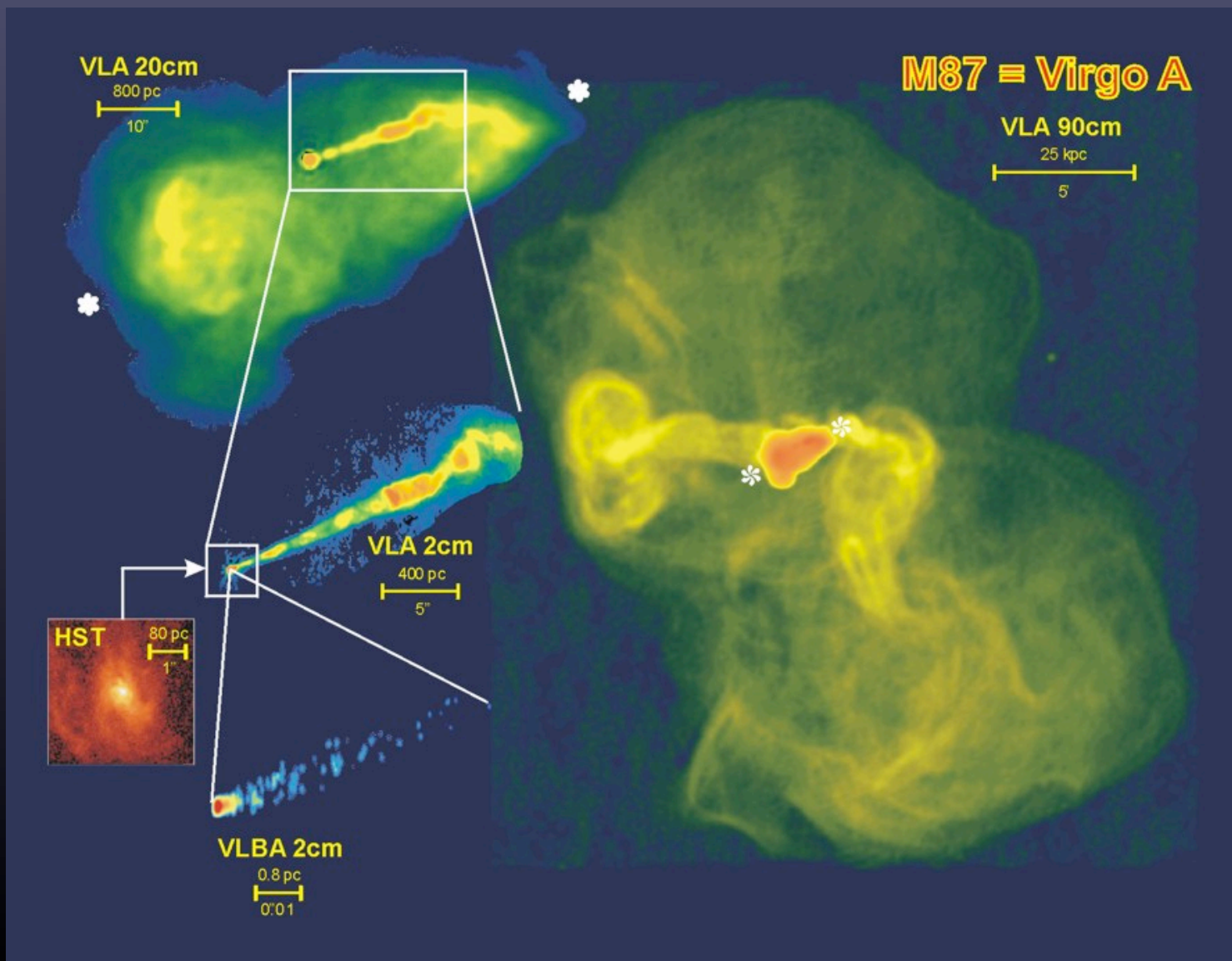


Jet Energy via Radio Bubbles in Hot Cluster Gas



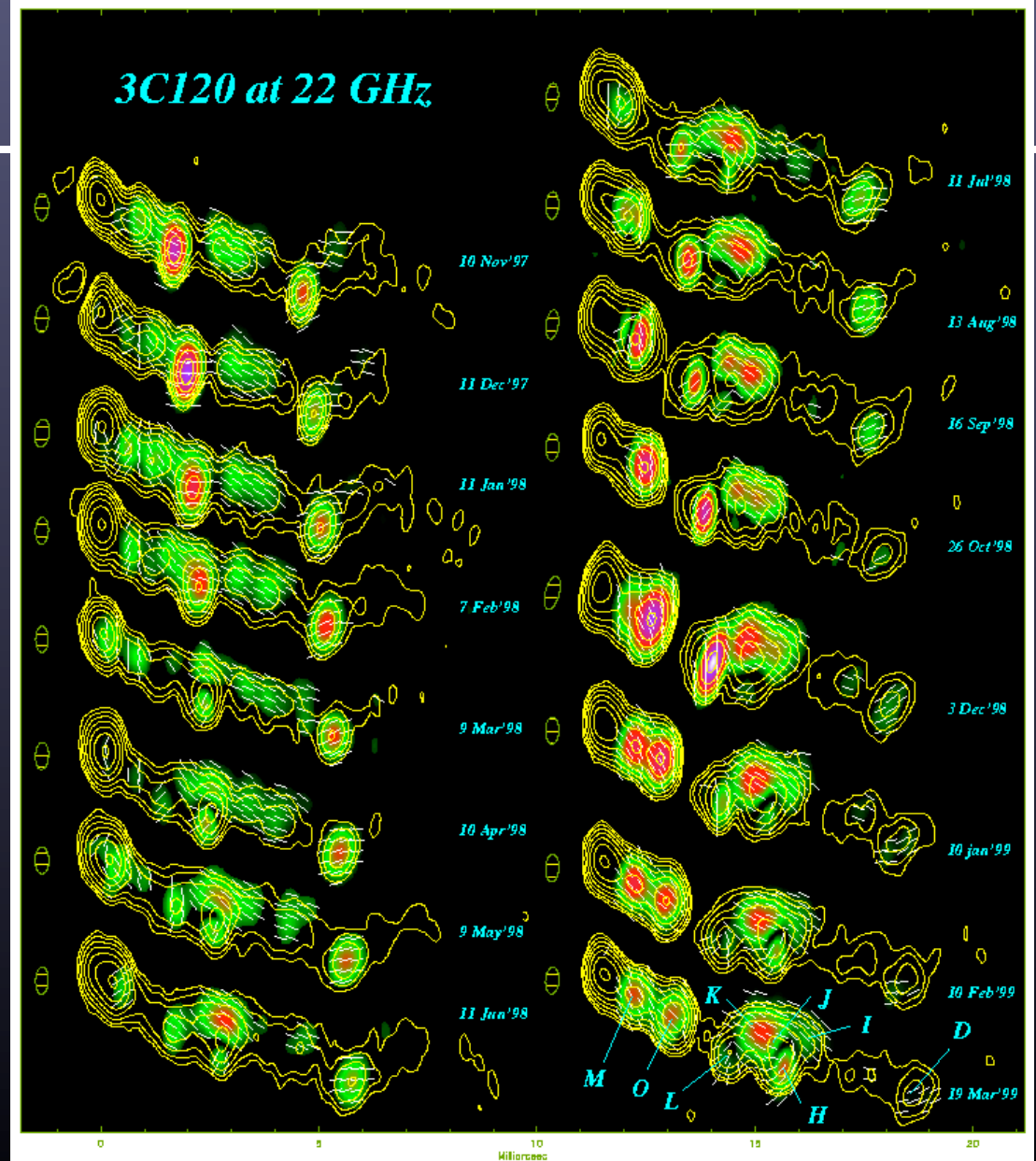
1.4 GHz VLA contours over Chandra X-ray image (left) and optical (right)
 6×10^{61} ergs $\sim 3 \times 10^7$ solar masses $\times c^2$ (McNamara et al. 2005, Nature, 433, 45)

Resolution and Surface-brightness Sensitivity



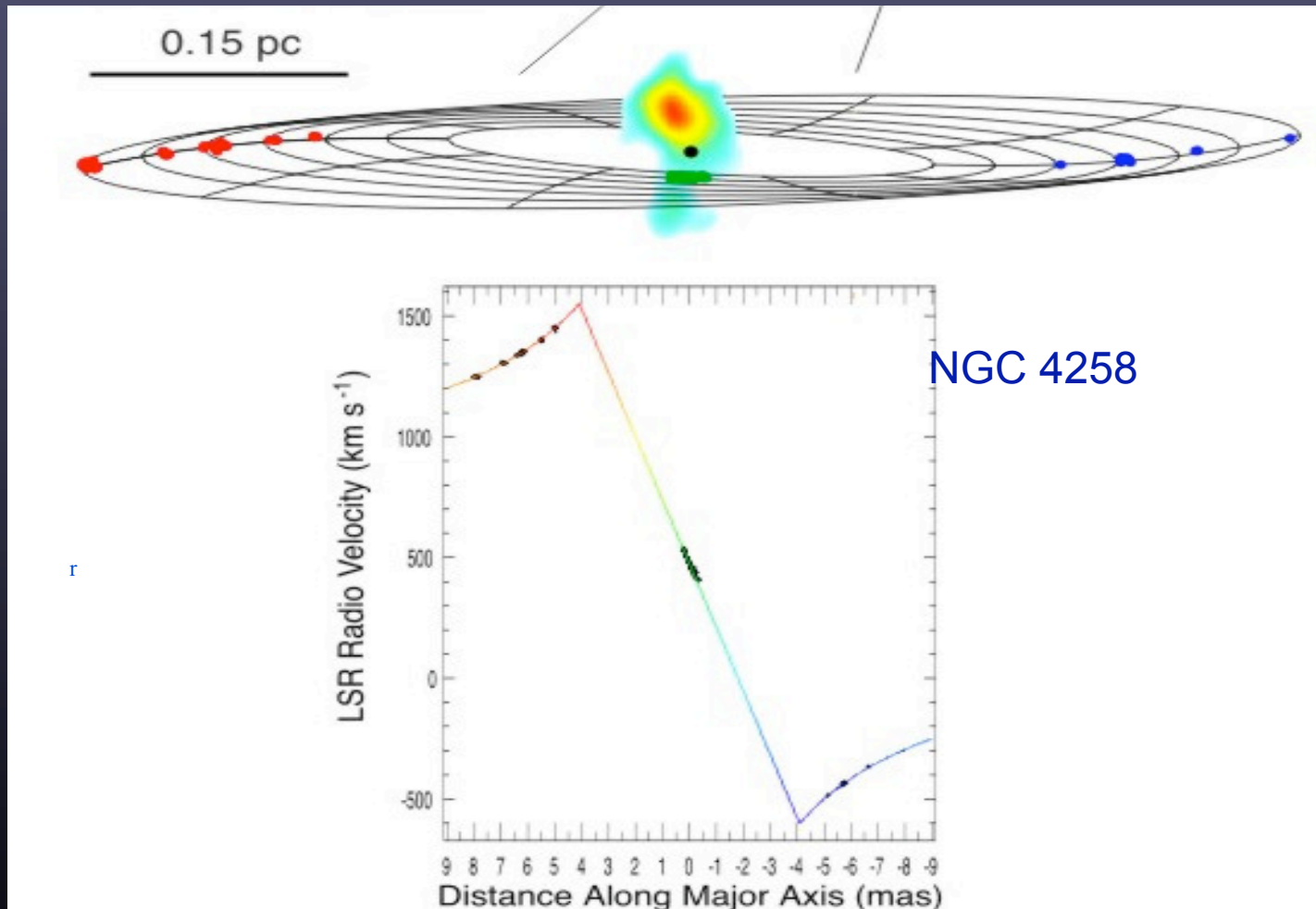
Superluminal Motion in Compact Jets

First major VLBI discovery: apparent superluminal motion, implying compact jets moving highly relativistically.



Resolving the circumnuclear disk in NGC 4258 and directly measuring the black-hole mass

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H₂O masers: get speed, period, positional accuracy (10 microarcseconds!)
directly measures SMBH masses, proper motions, acceleration, encl. density

Types of Antennas

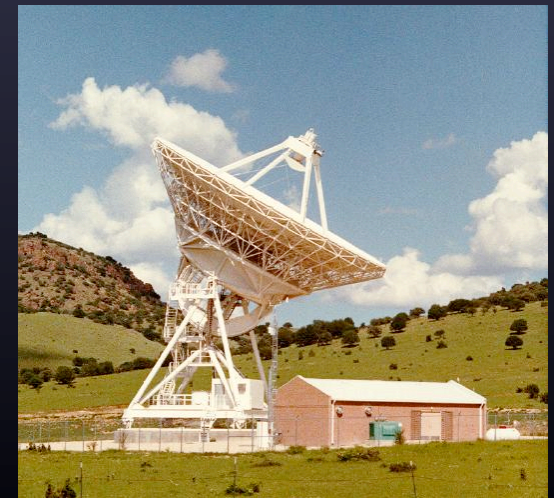
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- Wire antennas ($\lambda > 1\text{m}$)
 - Dipole
 - Yagi
 - Helix
 - Small arrays of the above
- Reflector antennas ($\lambda < 1\text{m}$)
- Hybrid antennas ($\lambda \approx 1\text{m}$)
 - Wire reflectors
 - Reflectors with dipole feeds

Yagi



Helix



Antennas – the Single Dish

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- The simplest radio telescope (other than elemental devices such as a dipole or horn) is a parabolic reflector – a ‘single dish’ with associated feed(s).
- Four important characteristics of an antenna:
 - They have a directional gain.
 - They have an angular resolution given by: $\theta \sim \lambda/D$.
 - They have ‘sidelobes’ – finite response at large angles.
 - Their angular response contains no sharp edges.
- A basic understanding of the origin of these characteristics will aid in understanding the functioning of an interferometer.

Basic Antenna Formulas

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Effective collecting area $A(\nu, \theta, \phi)$ m²

On-axis response $A_0 = \eta A$
 η = aperture efficiency

Normalized pattern (primary beam)

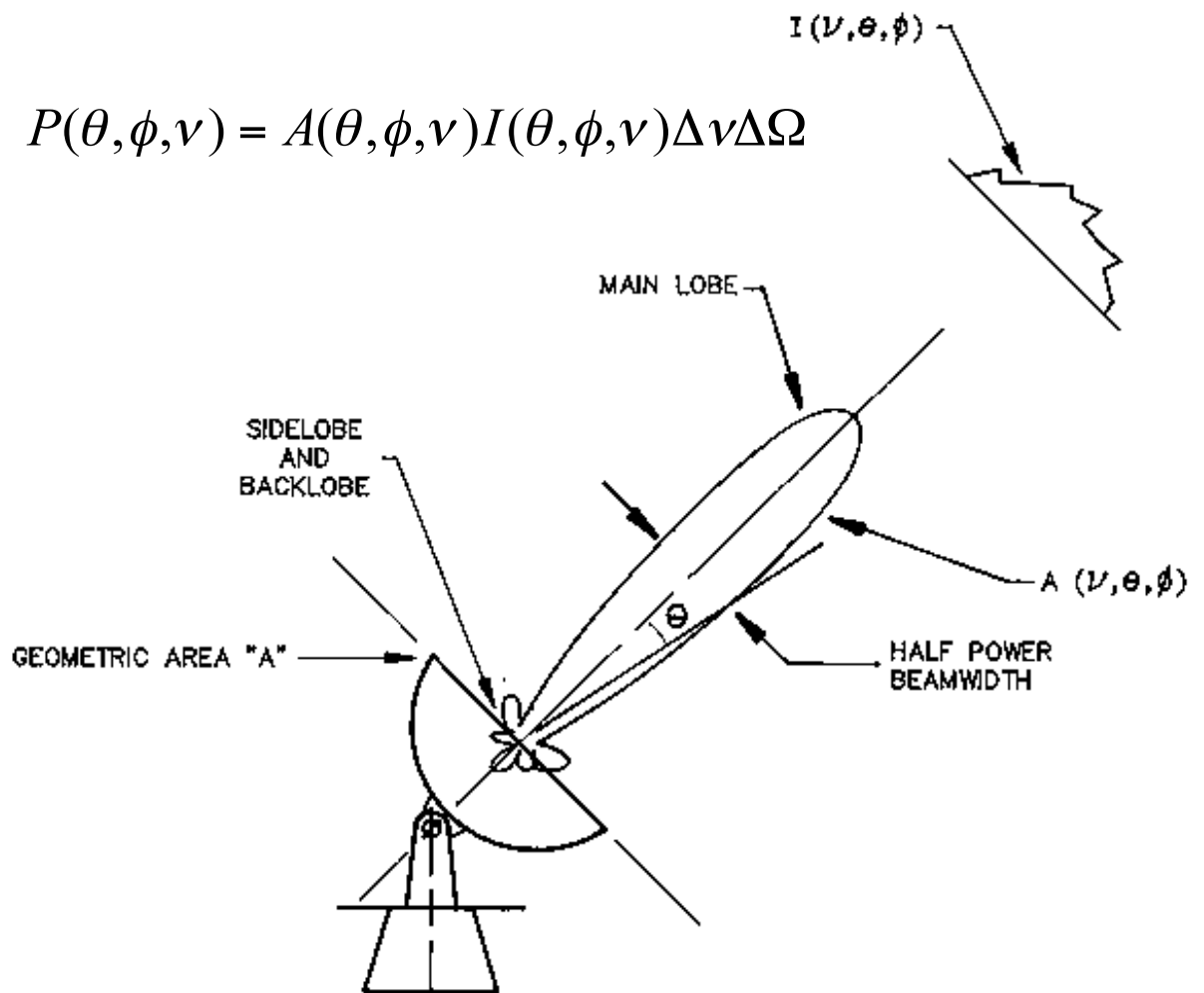
$$A(\nu, \theta, \phi) = A(\nu, \theta, \phi) / A_0$$

Beam solid angle

$$\Omega_A = \iint_{\text{all sky}} A(\nu, \theta, \phi) d\Omega$$

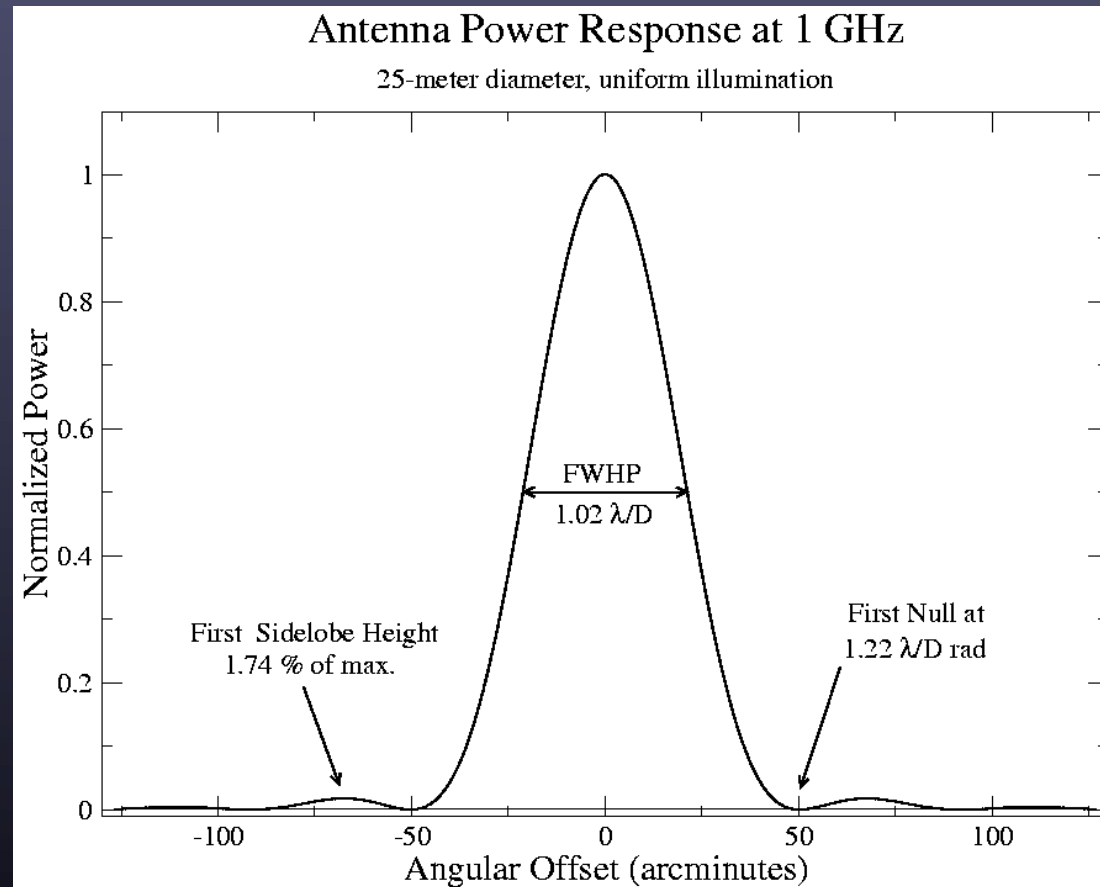
$$A_0 \Omega_A = \lambda^2$$

λ = wavelength, ν = frequency



The Standard Parabolic Antenna Response

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- “illumination” helps to determine this response
- This response important because you want to “clean” out emission from sidelobes and restore into your main beam (little different for an interferometer)

Reflector Optics: Examples

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Prime focus
(GMRT)



Cassegrain focus
(AT)



Offset Cassegrain
(VLA)



Naysmith
(OVRO)



Beam Waveguide
(NRO)

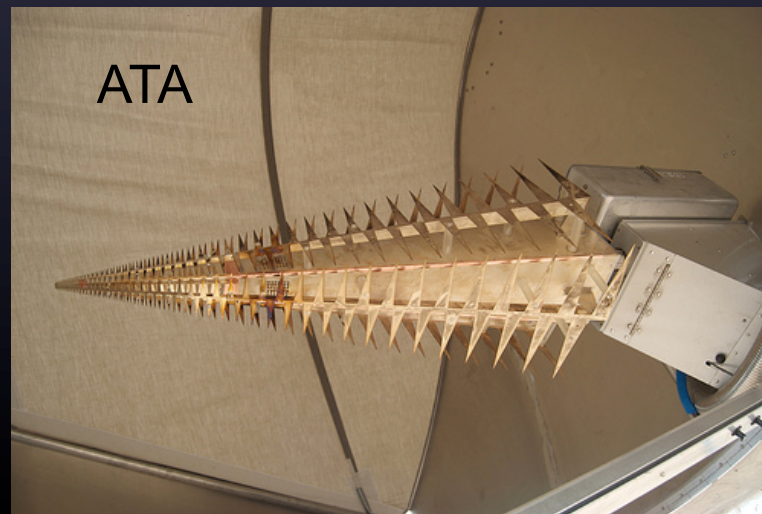
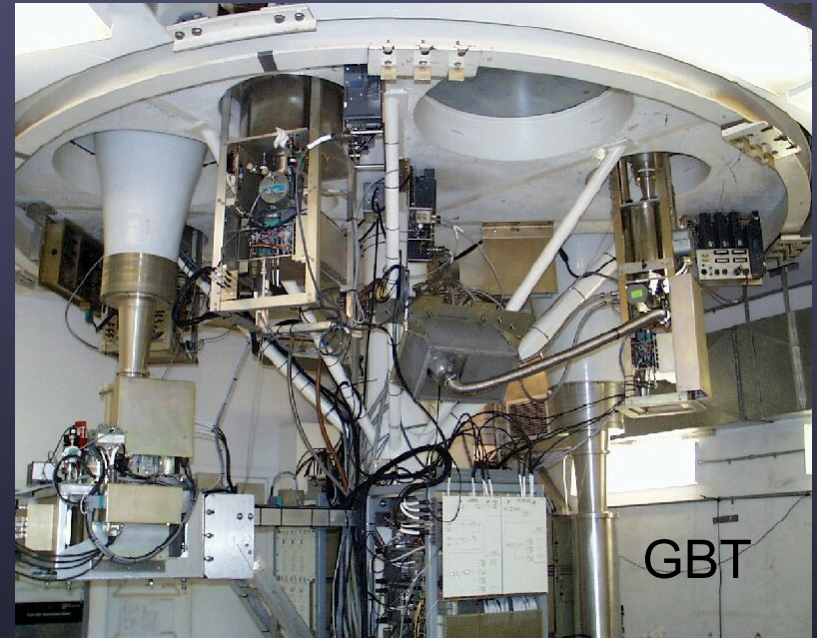


Dual Offset
(GBT)



Feed Systems

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Antenna Performance: Aperture Efficiency

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On axis response: $A_0 = \eta A$

Efficiency: $\eta = \eta_{sf} \times \eta_{bl} \times \eta_s \times \eta_t \times \eta_{misc}$

η_{sf} = Reflector surface efficiency

Due to imperfections in reflector surface

$$\eta_{sf} = \exp(-4\pi\sigma/\lambda)^2 \quad \text{e.g., } \sigma = \lambda/16, \eta_{sf} = 0.5$$

η_{bl} = Blockage efficiency

Caused by subreflector and its support structure

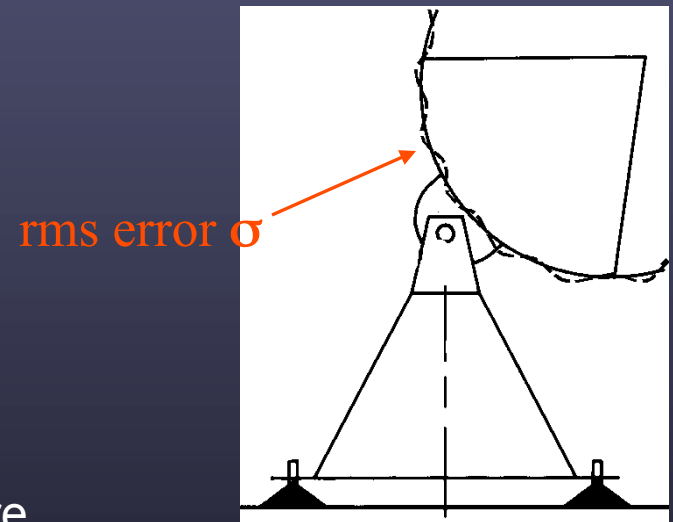
η_s = Feed spillover efficiency

Fraction of power radiated by feed intercepted by subreflector

η_t = Feed illumination efficiency

Outer parts of reflector illuminated at lower level than inner part

η_{misc} = Reflector diffraction, feed position phase errors, feed match and loss



VLA @ 4.8 GHz (C-band)

Interferometer Block Diagram

Key



Amplifier



Mixer



Correlator

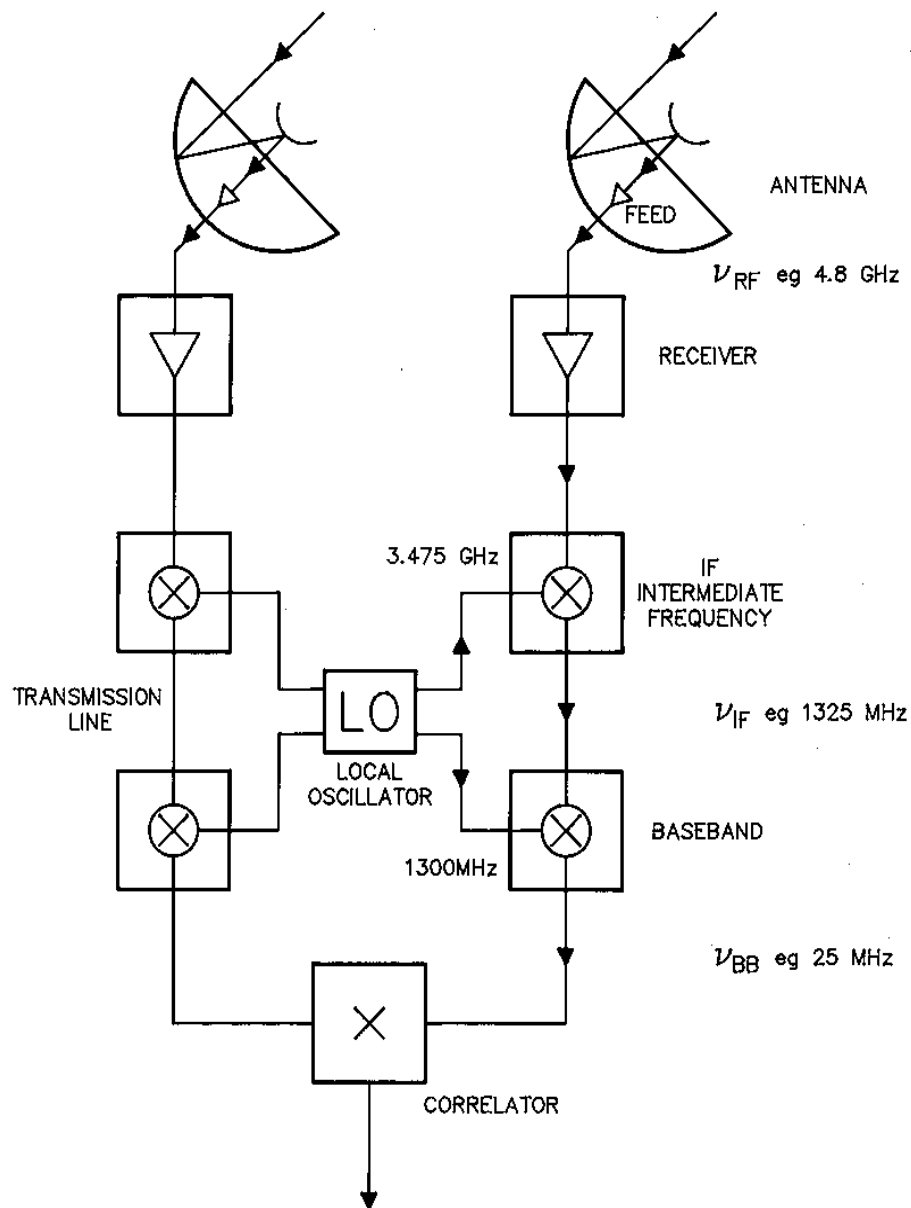
Antenna

Front End

IF

Back End

Correlator



The Polarization Ellipse

- From Maxwell's equations $\mathbf{E} \cdot \mathbf{B} = 0$ (E and B perpendicular)
 - By convention, we consider the time behavior of the E-field in a fixed perpendicular plane, from the point of view of the receiver.

- For a monochromatic wave of frequency ν , we write

$$E_x = A_x \cos(2\pi\nu t + \phi_x)$$

$$E_y = A_y \cos(2\pi\nu t + \phi_y)$$

- These two equations describe an ellipse in the (x-y) plane.
- The ellipse is described fully by three parameters:
 - A_x , A_y , and the phase difference, $\delta = \phi_y - \phi_x$.
- The wave is elliptically polarized. If the E-vector is:
 - Rotating clockwise, the wave is 'Left Elliptically Polarized',
 - Rotating counterclockwise, it is 'Right Elliptically Polarized'.

Stokes parameters

- Spherical coordinates: radius I , axes Q , U , V
 - $I = \sqrt{E_X^2 + E_Y^2} = \sqrt{E_R^2 + E_L^2}$
 - $Q = I \cos 2\chi \cos 2\psi = E_X^2 - E_Y^2 = 2 E_R E_L \cos \delta_{RL}$
 - $U = I \cos 2\chi \sin 2\psi = 2 E_X E_Y \cos \delta_{XY} = 2 E_R E_L \sin \delta_{RL}$
 - $V = I \sin 2\chi = 2 E_X E_Y \sin \delta_{XY} = E_R^2 - E_L^2$
- Only 3 independent parameters:
 - wave polarization confined to surface of Poincare sphere
 - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I, Q, U, V
 - defined by George Stokes (1852)
 - form complete description of wave polarization
 - **NOTE: above true for 100% polarized monochromatic wave!**

Linear Polarization

- Linearly Polarized Radiation: $V = 0$

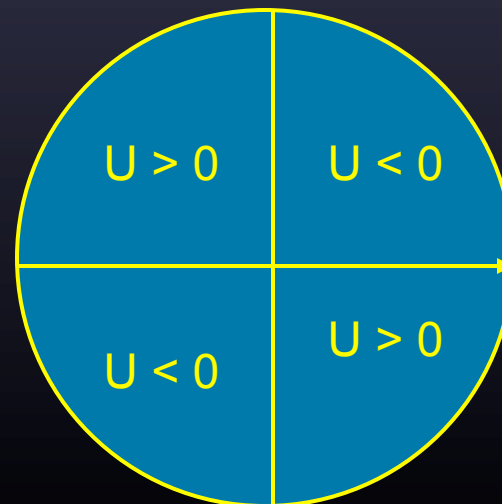
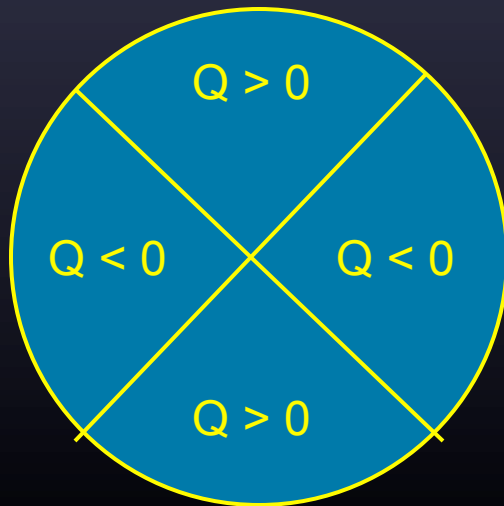
- Linearly polarized flux:

$$P = \sqrt{Q^2 + U^2}$$

- Q and U define the linear polarization position angle:

$$\tan 2\psi = U / Q$$

- Signs of Q and U:

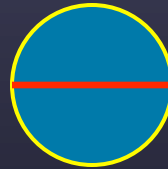


Simple Examples

- If $V = 0$, the wave is linearly polarized. Then,
 - If $U = 0$, and Q positive, then the wave is vertically polarized, $\Psi = 0^\circ$



- If $U = 0$, and Q negative, the wave is horizontally polarized, $\Psi = 90^\circ$



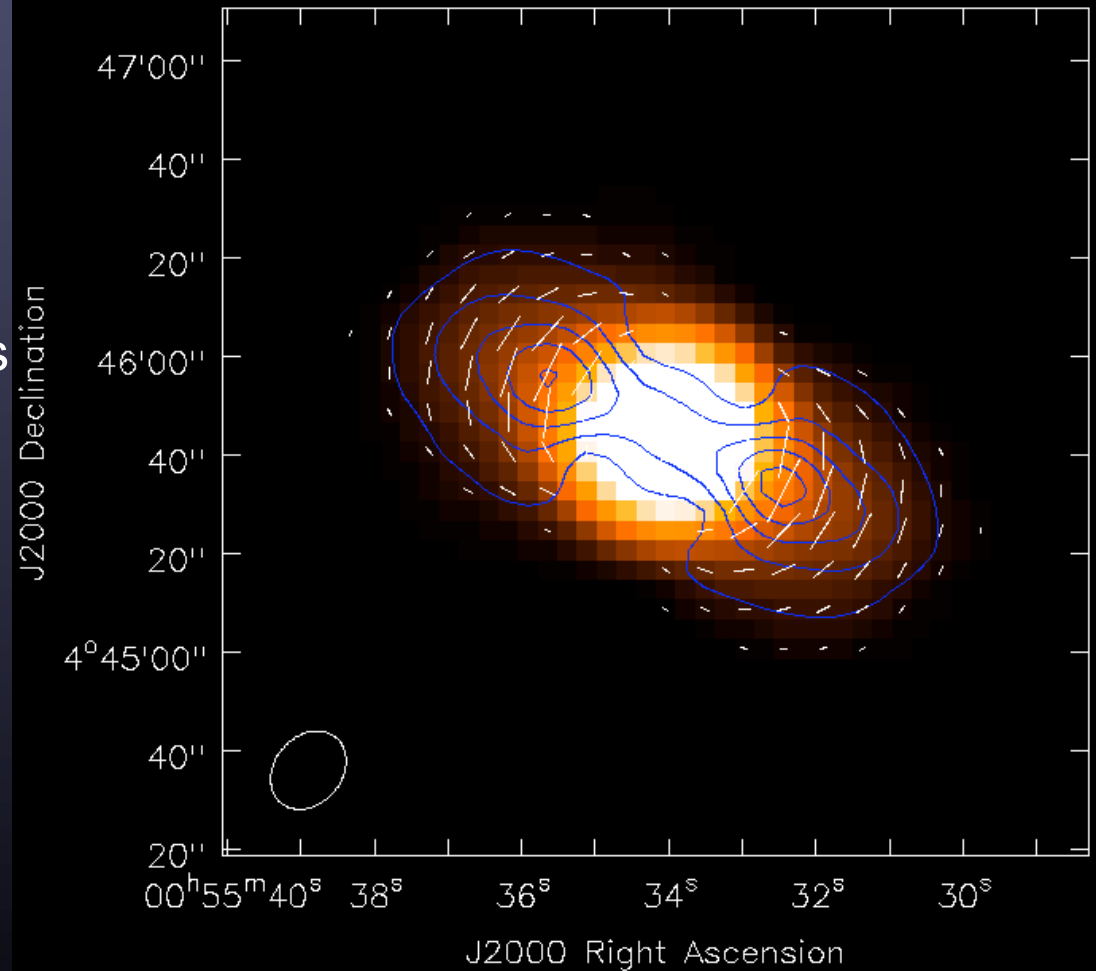
- If $Q = 0$, and U positive, the wave is polarized at $\Psi = 45^\circ$



- If $Q = 0$, and U negative, the wave is polarized at $\Psi = -45^\circ$.

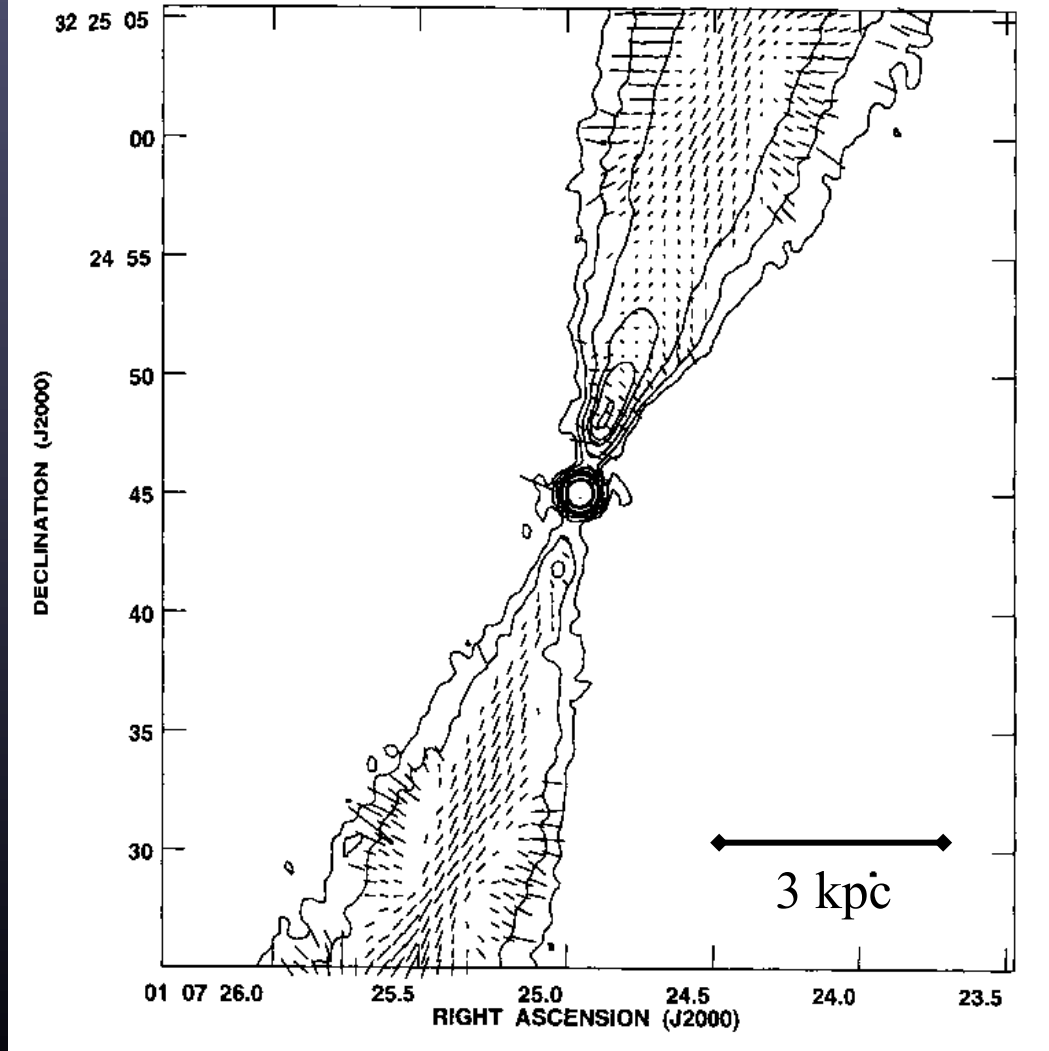


- Apr 1999 VLA 5 GHz data
- D-config resolution is $14''$
- Jupiter emits thermal radiation from atmosphere, plus polarized synchrotron radiation from particles in its magnetic field
- Shown is the I image (intensity) with polarization vectors rotated by 90° (to show B-vectors) and polarized intensity (blue contours)
- The polarization vectors trace Jupiter's dipole
- Polarized intensity linked to the Io plasma torus

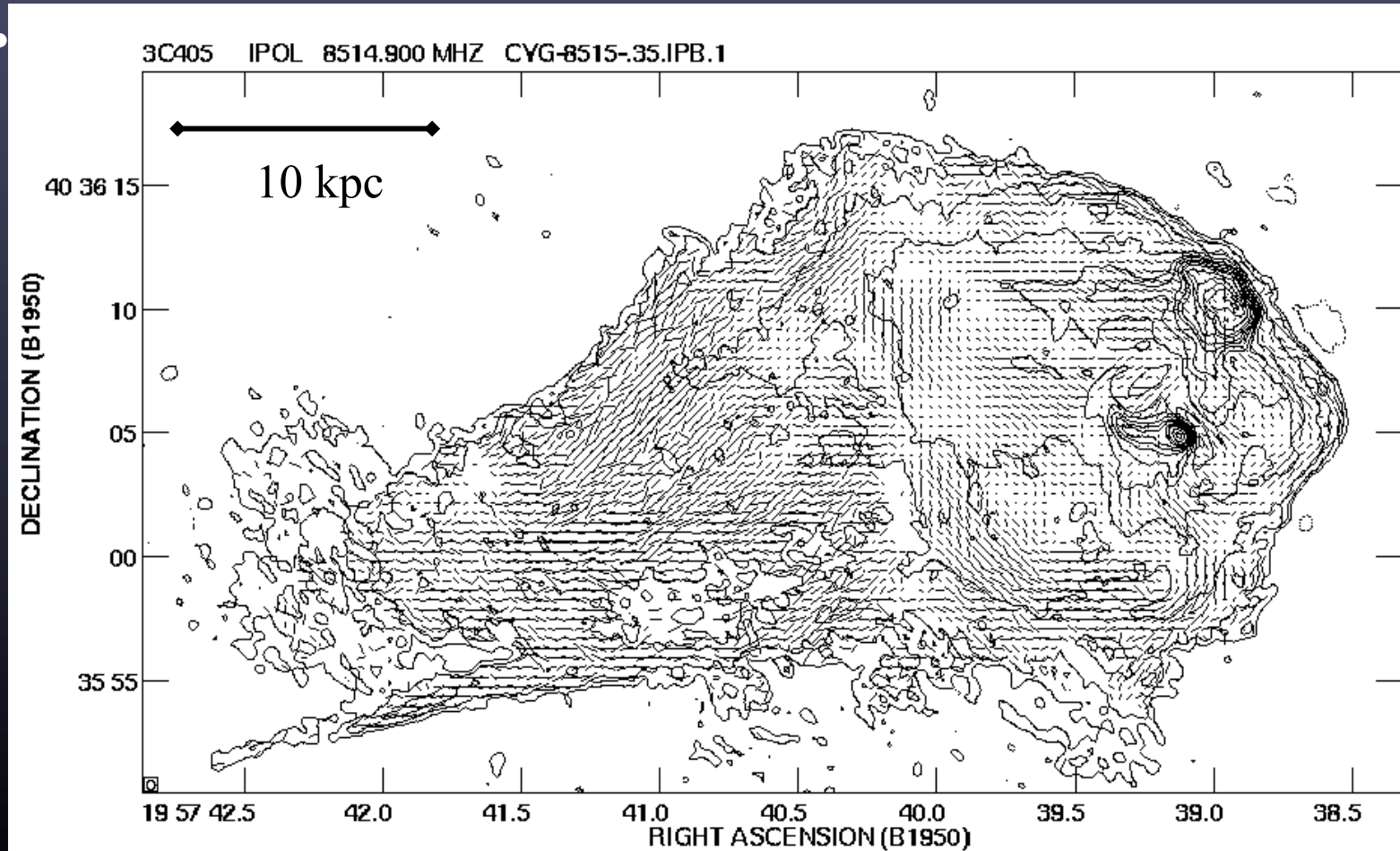


Example: Radio Galaxy 3C31

- VLA @ 8.4 GHz
- E-vectors
 - along core of jet
 - radial to jet at edge
- Laing (1996)



Example: Radio Galaxy Cygnus A



Getting Better Resolution: Interferometry

- The 25-meter aperture of a VLA antenna provides insufficient resolution for modern astronomy.
 - 30 arcminutes at 1.4 GHz, when we want 1 arcsecond or better!
- The trivial solution of building a bigger telescope is not practical. 1 arcsecond resolution at $\lambda = 20$ cm requires a 40 kilometer aperture.
 - The world's largest fully steerable antenna (operated by the NRAO at Green Bank, WV) has an aperture of only 100 meters \Rightarrow 4 times better resolution than a VLA antenna.
- As this is not practical, we must consider a means of synthesizing the equivalent aperture, through combinations of elements.
- This method, termed 'aperture synthesis', was developed in the 1950s in England and Australia. Martin Ryle (University of Cambridge) earned a Nobel Prize for his contributions.

Establishing Some Basics

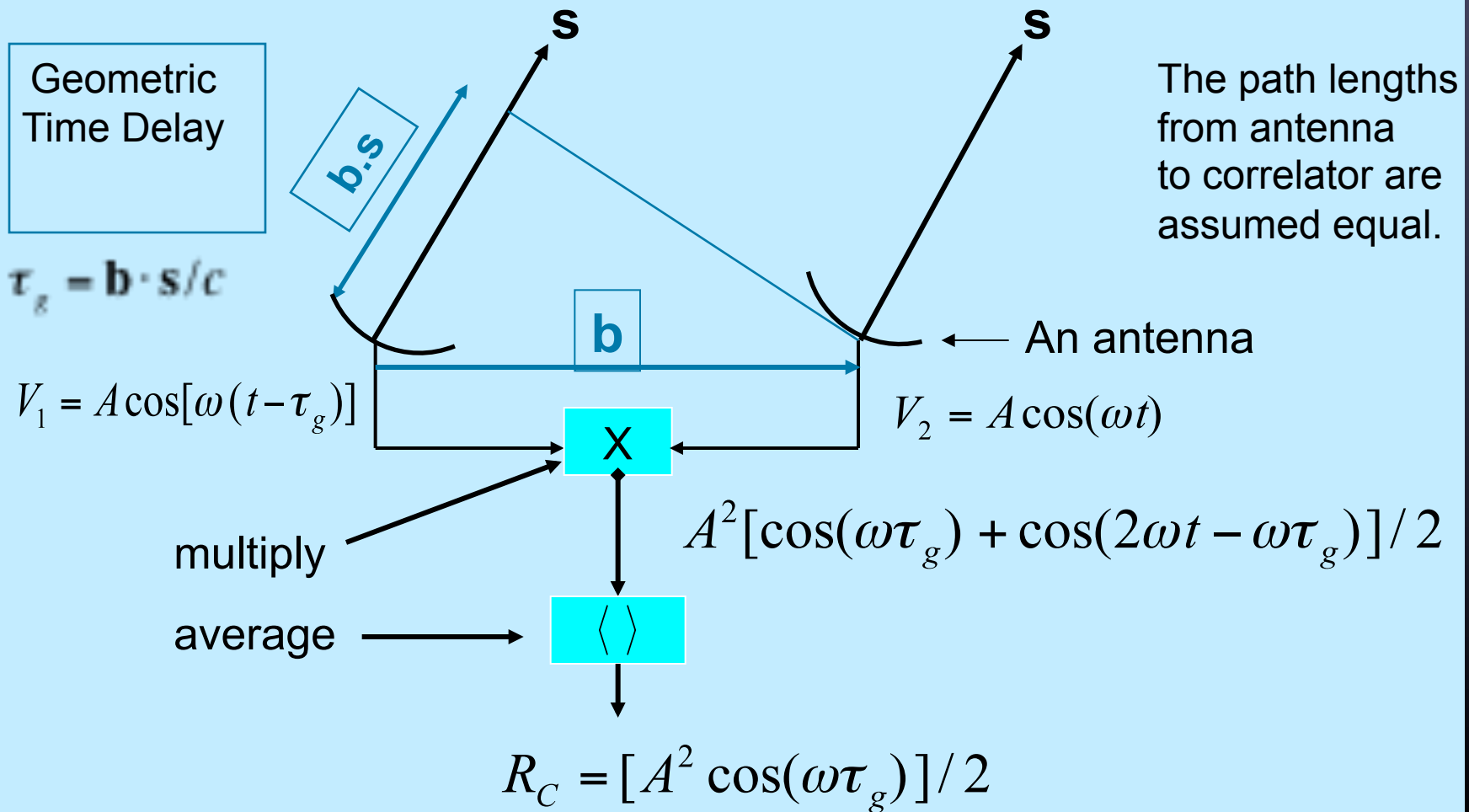
- Consider radiation from direction s from a small elemental solid angle, $d\Omega$, at frequency ν within a frequency slice, $d\nu$.
- For sufficiently small $d\nu$, the electric field properties (amplitude, phase) are stationary over timescales of interest (seconds), and we can write the field as

$$E_\nu(t) = A \cos(\omega t + \phi)$$

- The purpose of an antenna and its electronics is to convert this E-field to a voltage, $V(t)$ – proportional to the amplitude of the electric field, and which preserves the phase of the E-field – which can be conveyed from the collection point to some other place for processing.
- We ignore the gain of the electronics and the collecting area of the antennas – these are calibratable items ('details').
- The coherence characteristics can be analyzed through consideration of the dependencies of the product of the voltages from the two antennas.

The Stationary, Quasi-Monochromatic Interferometer

- Consider radiation from a small solid angle $d\Omega$, from direction \mathbf{s} , at frequency ν , within $d\nu$:



Examples of the Signal Multiplications

The two input voltages are shown in red and blue, their product is in black. The desired coherence is the average of the black trace.

In Phase: $\omega\tau_g = 2\pi n$

$$R_C = A^2 / 2$$

Quadrature Phase:

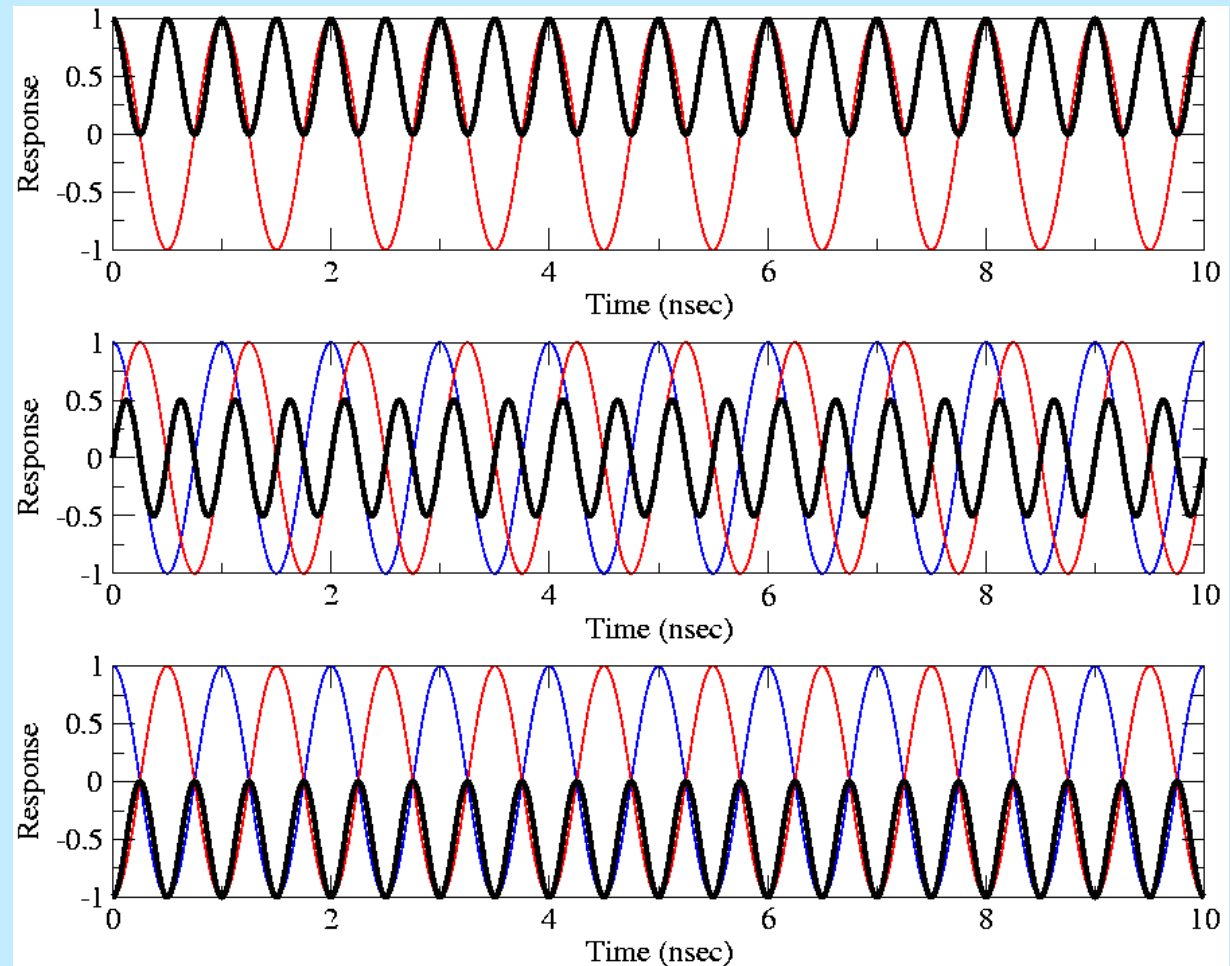
$\omega\tau_g = (2n+1)\pi/2$

$$R_C = 0$$

Anti-Phase:

$\omega\tau_g = (2n+1)\pi$

$$R_C = -A^2 / 2$$



Signal Multiplication, cont.

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- The averaged product R_C is dependent on the source power, A^2 and geometric delay, τ_g :

$$\omega\tau_g = 2\pi\nu\mathbf{b} \cdot \mathbf{s}/c = 2\pi\mathbf{b} \cdot \mathbf{s}/\lambda$$

- R_C is thus dependent only on the source strength, location, and baseline geometry.
- R_C is not a function of:
 - The time of the observation (provided the source itself is not variable!)
 - The location of the baseline, provided the emission is in the far-field.
- The strength of the product is also dependent on the antenna areas and electronic gains – but these factors can be calibrated for.
- We identify the product A^2 with the specific intensity (or brightness) I_ν of the source within the solid angle $d\Omega$ and frequency slice $d\nu$.

The Response from an Extended Source

- The response from an extended source is obtained by summing the responses for each antenna over the sky, multiplying, and averaging:

$$R_C = \left\langle \int V_1 d\Omega_1 \int V_2 d\Omega_2 \right\rangle$$

- The expectation, and integrals can be interchanged, and **providing the emission is spatially incoherent**, we get

$$R_C = \int \int I_\nu(\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

- This expression links what we want – the source brightness on the sky, $I_\nu(\mathbf{s})$, – to something we can measure - R_C , the interferometer response.

A Schematic Illustration of Correlation

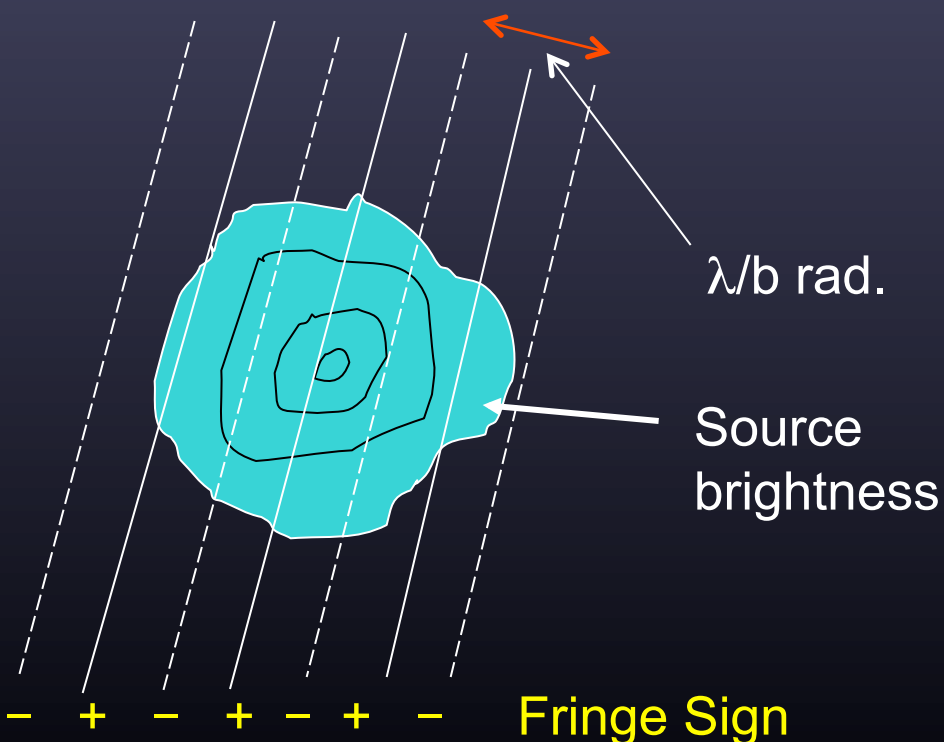
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- The correlator can be thought of 'casting' a sinusoidal coherence pattern, of angular scale λ/b radians, onto the sky.
- The correlator multiplies the source brightness by this coherence pattern, and integrates (sums) the result over the sky.

- Orientation set by baseline geometry.
- Fringe separation set by (projected) baseline length and wavelength.

- Long baseline gives close-packed fringes
- Short baseline gives widely-separated fringes

- Physical location of baseline unimportant, provided source is in the far field.

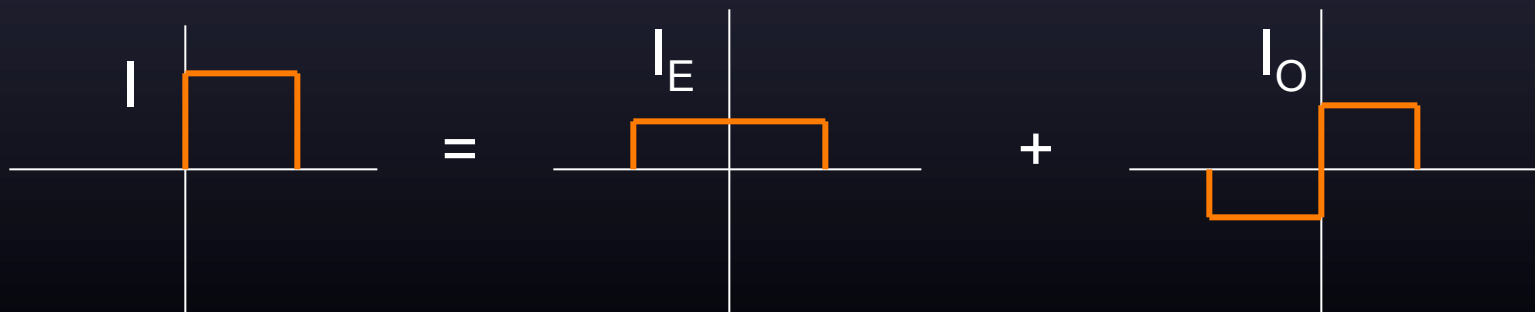


Odd and Even Functions

- But the measured quantity, R_c , is insufficient – it is only sensitive to the ‘even’ part of the brightness, $I_E(\mathbf{s})$.
- Any real function, I , can be expressed as the sum of two real functions which have specific symmetries:

An even part: $I_E(x,y) = (I(x,y) + I(-x,-y))/2 = I_E(-x,-y)$

An odd part: $I_O(x,y) = (I(x,y) - I(-x,-y))/2 = -I_O(-x,-y)$



Recovering the 'Odd' Part: The SIN Correlator

The integration of the cosine response, R_C , over the source brightness is sensitive to only the even part of the brightness:

$$R_C = \int \int (\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega = \int \int_E (\mathbf{s}) \cos(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

since the integral of an odd function (I_O) with an even function ($\cos x$) is zero.

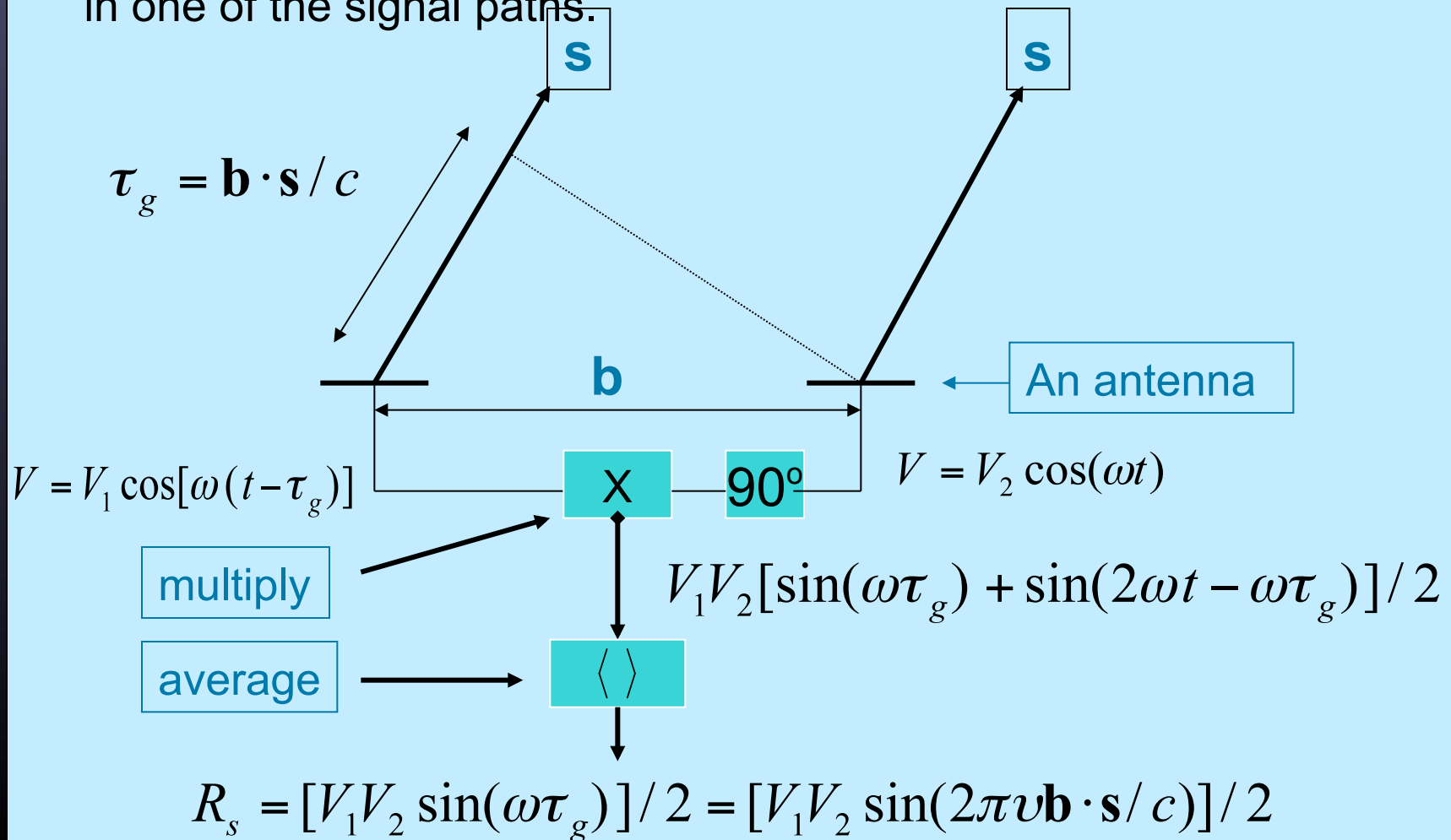
To recover the 'odd' part of the intensity, I_O , we need an 'odd' coherence pattern. Let us replace the 'cos' with 'sin' in the integral:

$$R_S = \int \int (\mathbf{s}) \sin(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega = \int \int_O (\mathbf{s}) \sin(2\pi \nu \mathbf{b} \cdot \mathbf{s} / c) d\Omega$$

since the integral of an even times an odd function is zero. To obtain this necessary component, we must make a 'sine' pattern.

Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.



Define the Complex Visibility

We now DEFINE a complex function, V , to be the complex sum of the two independent correlator outputs:

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:

$$V(\mathbf{b}) = R_C - iR_S = \int \int I_v(s) e^{-2\pi i \mathbf{v} \cdot \mathbf{s} / c} d\Omega$$

Although it may not be obvious (yet), this expression can be inverted to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

Making Images

We have shown that under certain (and attainable) assumptions about electronic linearity and narrow bandwidth, a complex interferometer measures the visibility, or complex coherence:

$$V_v(u, v) = \iint \frac{I_v(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2i\pi(ul + vm)} dl dm$$

(u, v) are the projected baseline coordinates, measured in wavelengths, on a plane oriented facing the phase center, and

(l, m) are the sines of the angles between the phase center and the emission, in the EW and NS directions, respectively.

Making Images

This is a Fourier transform relation, and it can be in general be solved, to give:

$$I_v(l, m) = \cos(\gamma) \iint V(u, v) e^{+2i\pi(ul+vm)} du dv$$

This relationship presumes knowledge of $V(u, v)$ for all values of u and v . In fact, we have a finite number, N , measures of the visibility, so to obtain an image, the integrals are replaced with a sum:

$$I_v(l, m) = \frac{1}{N} \sum_{n=1}^N V_n(u_n, v_n) \exp[2\pi i(u_n l + v_n m)] \Delta u \Delta v$$

If we have N_v visibilities, and N_m cells in the image, we have $\sim N_v N_m$ calculations to perform – a number that can exceed 10^{12} !

VLA @ 4.8 GHz (C-band)

Interferometer Block Diagram

Key



Amplifier



Mixer



Correlator

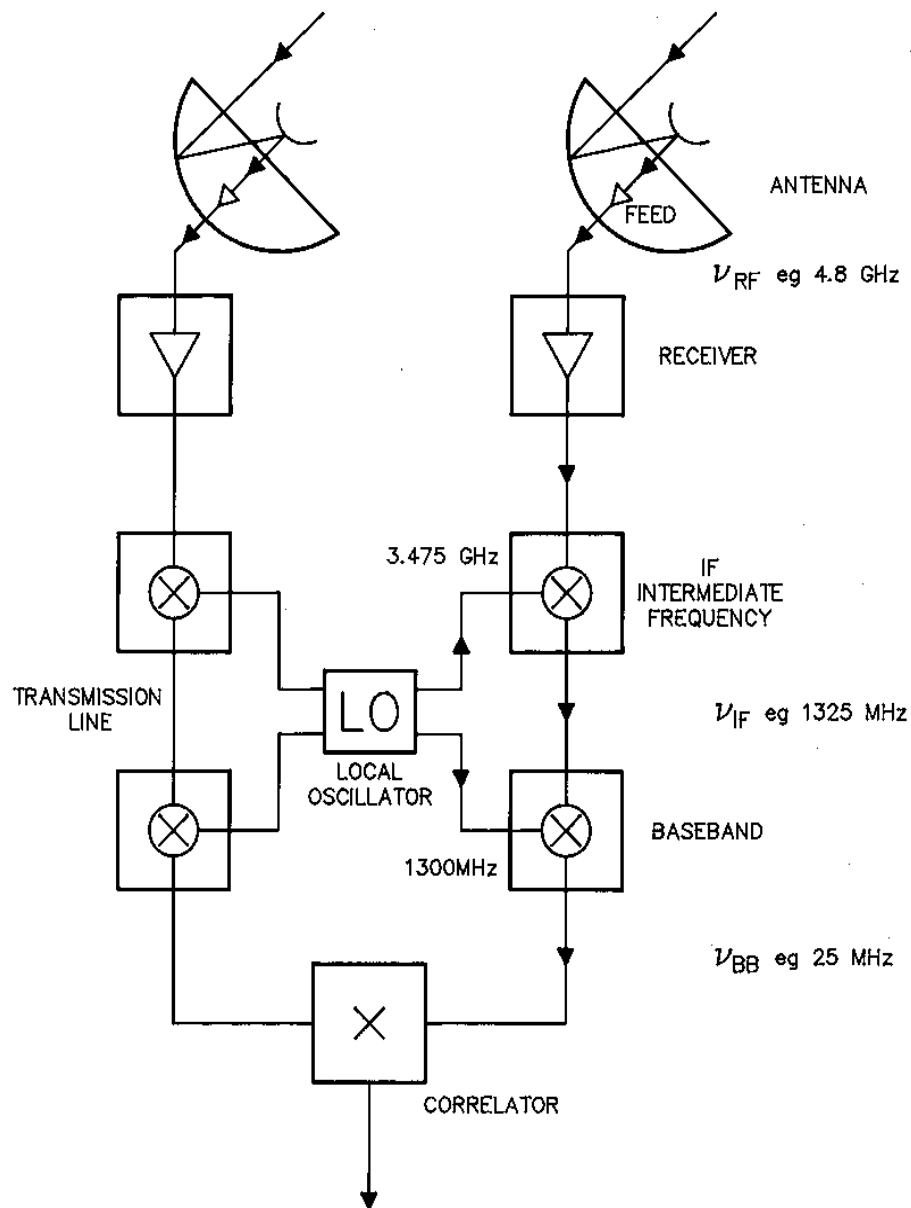
Antenna

Front End

IF

Back End

Correlator



Importance of Antennas for Interferometers

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- Antenna amplitude pattern causes amplitude to vary across the source.
- Antenna phase pattern causes phase to vary across the source.
- Polarization properties of the antenna modify the apparent polarization of the source.
- Antenna pointing errors can cause time varying amplitude and phase errors.
- Variation in noise pickup from the ground can cause time variable amplitude errors.
- Deformations of the antenna surface can cause amplitude and phase errors, especially at short wavelengths.

Some radio astronomy definitions

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I_ν (or B_ν) = Surface Brightness : $\text{erg/s/cm}^2/\text{Hz/sr}$
(= intensity)

S_ν = Flux density : $\text{erg/s/cm}^2/\text{Hz} \int I_\nu \Delta\Omega$

S = Flux : $\text{erg/s/cm}^2 \int I_\nu \Delta\Omega \Delta\nu$

P = Power received : $\text{erg/s} \int I_\nu \Delta\Omega \Delta\nu \Delta A_{\text{tel}}$

E = Energy : $\text{erg} \int I_\nu \Delta\Omega \Delta\nu \Delta A_{\text{tel}} \Delta t$

Interferometric Radiometer Equation

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$$S_{rms} = \frac{2kT_{sys}}{A_{eff} \sqrt{N_A (N_A - 1) t_{int} \Delta\nu}}$$

- T_{sys} = wave noise for photons (RJ): rms \propto total power
- A_{eff}, k_B = Johnson-Nyquist noise + antenna temp definition
- $t\Delta\nu$ = # independent measurements of T_A/T_{sys} per pair of antennas
- N_A = # indep. meas. for array, or can be folded into A_{eff}

Summary

- **Radio Interferometry: a powerful tool**
 - Physical insight into many different processes
 - Spatial scales comparable or better than at other wavelengths: multi-wavelength approach
- **A great time for students & interferometry!**
 - Amazing science opportunities with new tools

